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A DIAGNOSTIC STUDY OF CERTAIN DIFFICULTIES WHICH INTERFERE WITH PUPIL PROGRESS IN SENIOR-HIGH-SCHOOL MATHEMATICS

Service Paper

Submitted by

Margaret Mary Scully

(A.B., College of New Rochelle, 1924)

In partial fulfillment of requirements for the degree of Master of Education

1946

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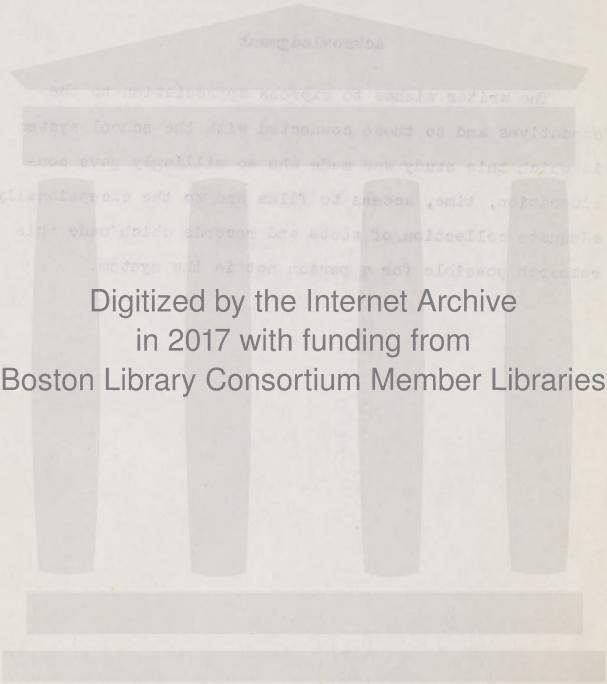
The writer wishes to express appreciation to the executives and to those connected with the school system in which this study was made who so willingly gave consideration, time, access to files and to the exceptionally adequate collection of tests and records which made this research possible for a person not in the system.



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CHAPTER I

PROBLEM AND METHOD

This study has been an attempt to diagnose certain difficulties that interfere with pupil progress in senior-high-school mathematics. The difficulties diagnosed were those which have persisted through successive mathematics courses from grade VI to grade XI and which could be detected by an analysis of the difficulties on standard tests which had already been given to check the achievement in those grades.

The writer has not attempted a detailed diagnosis in the weaknesses in fundamental processes because previous research has shown that various degrees of this deficiency exist among students through grade XII. Furthermore, material is available to aid in diagnosing and correcting such deficiencies. Publishing houses have good tests for diagnosing difficulties in the fundamental processes in arithmetic in both whole numbers and fractions. There are many text books that offer methods for correction in these weaknesses when they are recognized.

The writer has tried to diagnose and analyze some of the deficiencies in fundamental mathematical concepts that have persisted through elementary school and interfere with satisfactory progress in mathematics in high school. There

CHAPTER I

PROBLEM AND METHOD

was available for analysis six standard tests that had been given in grades eight, nine, and ten to students who at present make up the membership of mathematics 11 classes in a certain better-than-average high school in the vicinity of Boston which will be referred to as High School X.

Since there are people of above average intelligence whose ability in mathematics is inadequate, the writer believed that an analysis of the mathematical concepts or the lack of them in such persons would show up the ideas that need immediate stress in the teaching of elementary arithmetic. In order to select students of this type from grade 11 in High School X for case studies, a comparison of scores on certain standard tests with teachers' marks in mathematics 9 and mathematics 10 for individual students was made. The percentile rank of the total intelligence quotient of each pupil obtained from the Psychological Examination was used to divide the pupils of mathematics 11 classes into five groups. Group I consisted of those whose percentile ranks ranged from 80 to 100; group II, from 60 to 79; group III, from 40 to 59; group IV, from 20 to 39; and group V, 0 to 19.

Next, tables and charts were made to show the distribution in each of the five groups of A's, B's, C's, and X's earned by these pupils in the mathematics 9 and the mathematics 10 courses. In the same way, pupils of mathematics 11 were divided into five groups according to the percentile

ranks of their scores earned on the Cooperative Elementary Algebra Test, and a distribution of marks earned in mathematics 9 was made for each of these groups. The same kind of division of pupils in mathematics 11 classes into five groups according to their percentile ratings on the Cooperative General Mathematics Test was made, and the distribution of marks in mathematics 10 for the pupils in these groups was shown.

The writer has assumed that those in group I for any of the standard tests mentioned should earn A in the two mathematics courses considered; those in group II, B; those in group III, C; and those in group IV, X. There were no failure marks for pupils of mathematics 11 classes in mathematics 9 or mathematics 10 because passing work in these courses is a prerequisite for entrance into mathematics 11. Whenever a pupil's marks for mathematics 9 or mathematics 10 differed by two groups from that group in which his score on one of the standard tests placed him, he was considered a case suspicious of educational maladjustment. There were four such large scale comparisons made, and six pupils were discovered whose marks in mathematics 9 and mathematics 10 were in three instances two groups below where their scores on standard tests placed them. These six pupils were selected for case studies.

The case studies consisted of a diagnosis of the

The case studies consisted of a diagnosis of the

difficulties found in three standard mathematics tests, and in the sections dealing with mathematics or mathematical reasoning, which were included in a standard psychology test and in a standard test of mental maturity. Attitudes toward mathematics were analyzed from the results of a standard occupational interest inventory test and from responses made on a questionnaire devised by the writer. The writer does not know personally the pupils selected, and has been handicapped by not having the opportunity to observe them at work on mathematics or in their ways of daily living. The conclusions have been based upon the writer's interpretation of pupil difficulties found on the standard tests which were analyzed. The tests are here listed with the grade in which each was administered and the date.

Iowa Every-Pupil Tests of Basic Skills, Advanced Battery-Grades 5, 6, 7, and 8 Form M, Test D: Basic Arithmetic Skills, H. F. Spitzer, State University of Iowa, Houghton Mifflin Company, Boston, 1941.
(Administered in grade 8, May 5, 1943)

American Council on Education Psychological Examination for High School Students, L. L. Thurston and T.G. Thurston, American Council on Education, 744 Jackson Place, Washington, D.C., 1942.
(Administered in grade 9, March, 1944.)

American Council on Education Cooperative Algebra Test, Elementary Algebra Through Quadratics, Revised Series Form R, J. A. Long, L. P. Siceloff, and L. E. Chesire, The Cooperative Test Service, 15 Amsterdam Avenue, New York City. 1941
(Administered in grade 9, May 18, 1944.)

Occupational Interest Inventory Intermediate, Form A, E. A. Lee and L. P. Thorpe, California Test Bureau, 5916 Hollywood Boulevard, Los Angeles 28, California, 1944. (Administered in grade 10, December, 1944.)

California Short-Form Test of Mental Maturity Advanced S-Form, E. T. Sullivan, W. W. Clark, and E. W. Tiegs, California Test Bureau, 5916 Hollywood Boulevard, Los Angeles 28, California, 1942.
(Administered in grade 10, January 29, 1945)

American Council on Education Cooperative General Mathematics Test for High School Classes, Revised Series Form O, H. T. Lundholm and L. P. Siceloff, The Cooperative Test Service, 15 Amsterdam Avenue, New York City, 1938.
(Administered in grade 10, May 10, 1945.)

Questionnaire Concerning Attitudes Toward Mathematics, devised by the writer.
(Given in grade 11, November 20, 1945.)

CHAPTER II

PREVIOUS RESEARCH

The use of tests to discover learning difficulties began about 1923.[1] During the following ten years studies analyzing difficulties in arithmetic and in problem solving, and tests to diagnose weaknesses in computational arithmetic grew in importance. Buswell,[2] one of the pioneers in diagnosis in education, published in 1929 a summary of educational investigations relating to arithmetic. The early studies in diagnosis were in the field of elementary education. In 1928 Klapp, Nason and Heilman[3] used diagnostic and remedial methods in reviewing the first three semesters of algebra in a class of college preparatory students. The description of this experiment was insufficient, so the results were not highly significant.

1/John L. Stenquist, "Development in Uses of Tests," Review of Educational Research (February, 1933), III: No 1, p. 57.

2/G. T. Buswell and C. H. Judd, Summary of Investigations Relative to Arithmetic, University of Chicago Press, Chicago, 1929.

3/ W. J. Klapp, L. J. Nason, and K. K. Heilman, "Diagnostic and Remedial Review of the First Three Semesters of Algebra," School Science and Mathematics (June, 1929), 28: 639-643.

MacRae and Uhl[1] (1932), in their regular classes, investigated the worth of diagnostic testing and remedial instruction. They noted the types of errors that persisted and shifted the pupils into homogeneous groups. The results were good but only indicative of what could be discovered by controlled experimentation. That same year, the College Entrance Examination Board published "A Study in Error" by C. C. Brigham[2] who made a detailed study of about 1000 individual test questions and reported the actual results obtained from these items.

Ramsay[3] thought that it was surprising that pupils could attain the rank of junior and senior in high school and lack so many of the basic skills in mathematics which they were expected to know in elementary school. With the help of diagnostic tests, he selected nine pupils who were poorest in reading comprehension and six who were poorest in the fundamentals of arithmetic and offered them five weeks of remedial work during the summer without credit. This experiment demonstrated for him that (1) with limited facilities, teachers need not hesitate to undertake diagnostic

1/M. MacRae and W. L. Uhl, "Types of Errors and Remedial Work in the Fundamental Processes of Algebra," Journal of Educational Research (September, 1932), 26: 12-21.

2/C. C. Brigham, A Study in Error, College Entrance Examination Board, New York, 1932, 384 pp.

3/C. H. Ramsay, "Diagnostic Testing and Remedial Teaching in Junior-Senior High School," Junior Senior High School Clearing House (December, 1934), 9: 232-234.

Meares and Uhl[1] (1936), in their regular classes, investigated the worth of diagnostic testing and remedial instruction. They noted the types of errors that persisted and applied the pupils into homogeneous groups. The results were good but only indicative of what could be discovered by controlled experimentation. That same year, the College Entrance Examination Board published "A Study in Error" by O. O. Brigham[2] who made a detailed study of about 1000 individual test questions and reported the actual results obtained from these items.

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1\. Meares and Uhl. Unt. "Types of Errors and Remedial Work in the Fundamental Processes of Algebra." Journal of Educational Research (September, 1936), 28: 12-27.

2\. O. Brigham. A Study in Error. College Entrance Examination Board. New York, 1932. 208 pp.

3\. H. Ramsay, "Diagnostic Testing and Remedial Teaching in Junior-Senior High School." Junior Senior High School Clearing House (December, 1934), 3: 228-234.

testing and remedial teaching; (2) that deficient pupils in a junior-senior high school can make large gains in ability to study, and in acquiring the fundamental concepts of arithmetic; and (3) that remedial instruction, as evaluated by teacher judgment, carries over into regular classroom instruction.

The Thirty-fourth Yearbook of the National Society for the Study of Education devoted six chapters to principles and techniques of educational diagnosis and one chapter to diagnosis in arithmetic. There is nothing specific about diagnosis in secondary school mathematics, but there are numerous applicable general principles such as this statement by Tyler:[1] "If educational diagnosis is to be a handmaiden to effective teaching, it is first essential that diagnosis be made in connection with worthy objectives of a good educational program."

Fawcett[2] made a contribution in the non-computational field in his study "The Nature of Proof" in which some of the characteristics of behavior involved in the understanding of the nature of a proof are illustrated.

> It is unfortunate that studies in this field have been directed all too frequently toward prevention of

1/ R. W. Tyler, "Characteristics of a Satisfactory Diagnosis," Thirty-fourth Yearbook, National Society for the Study of Education, Plimpton Press, Norwood, Mass., 1935.

2/ H. P. Fawcett, "Demonstrative Geometry and the Nature of Proof," Ohio Schools (October, 1936), 14:285.

> failure, usually among the lower capacity groups, rather than at the problem of discovery of practices and materials which can be used by those concerned with educational guidance in an attempt to cause students of superior capacity to achieve more in harmony with their capacity to achieve.

With this thought in mind, Newland,[1] in a controlled experiment, investigated the effect of a program of educational guidance on a group of academic problem pupils in a sophomore class in high school, and found that in the experimental group there were (1) an increase in quality credits, (2) fewer failures, and (3) no withdrawals from school.

In 1938 Buckingham[2] brought out the idea of the need for diagnosis as a continuous process throughout the entire learning activity. He made the following statement.

> When a teacher asks, 'How can I teach long division of fractions or problem solving?', her questions require answers that must be sought in distant places. They are seldom found in methods to be used at the moment. They rest upon a slow development of significance and meaning, perhaps over a period of several years. If pupils have not understood what they have been doing from the first with easy material, if no insight has aided them in learning, they cannot understand by any act of ours or theirs applied later to more difficult material. They must go back and build up the meaning and significance of numbers, whole and fractional, which they employ. Only then will they be likely to understand division of fractions. Only then, with varied experience in the use of numbers and a sense of their inner meaning, can we expect the insight which will ensure the ability to solve problems.

1/T. E. Newland and W. E. Ackley, "An Experimental Study of the Effect of Educational Guidance on a Selected Group of High-school Sophomores," Journal of Experimental Education (September, 1936), 5: 23-25.

2/B. R. Buckingham, "Significance, Meaning, Insight--These Three," Mathematics Teacher (January, 1938), 31: 24-30.

Washburne[1] (1939) strikes the same note in the article concerning primary grade origins of later remedial cases. He claims that

> Introducing arithmetic sooner than the children's experiential background and mental development indicate are advisable, results in confusion as well as dislike of arithmetic. Instead of arithmetic being a solid ground work on which one can build, instead of its giving the security that good substantial figures give us who know the meaning of those figures, the child wanders helplessly around amid a mass of separate, meaningless half-memories, trying to find the particular one that will satisfy the teacher. Then we say that he needs remedial help!

Such teaching is time wasted and leads to definitely detrimental results. Teachers should be prepared to use inventory check procedures to reveal pupil weaknesses, before, during and after a learning activity. Good testing means good teaching.[2]

By 1942 readiness tests were being used to discover the factors in the total personality of the learner that are likely to interfere with subsequent success in learning in a given area.[3] When the results of such a test are used for diagnostic purposes, its value for prediction purposes may be considerably reduced, since the correction of the

1/C. W. Washburne, "Primary Grade Origins of Later Remedial Cases," Childhood Education (September, 1939), 16: 29-33.

2/G. M. Wilson, M. B. Stone, and C. O. Dalrymple, Teaching the New Arithmetic, McGraw-Hill Book Company, New York and London, 1939.

3/L. J. Brueckner, "The Necessity for a Continuous Program of Diagnosis", Journal of Educational Research (February, 1942), 35: 459-461.

Washburne[1] (1930) advises the same note in the article concerning primary grade objective or labor remedial cases. He states that

> ... independent arithmetic seems then the children's psychological background and mental development indicate advisable, results in contusion as well as difficulties of arithmetic. Instead of arithmetic being a solid ground work on which one can build, instead of its giving the security that good mathematical figures give me who know the meaning of those figures, the child wanders helplessly around and a mass of nonsense, meaningless drill-memorized, trying to find the particular one that will satisfy the teacher. Then we say that he needs remedial help!

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1. W. Washburne, "Primary Grade Origins of Later Remedial Cases," Childhood Education (September, 1930), 16: 20-23.

2. M. William, K. R. Stone, and C. C. Dalrymple, Teaching The New Arithmetic. McGraw-Hill Book Company, New York and London, 1939.

3. I. Bemelmer, "The Necessity for a Continuous Program of Diagnosis", Journal of Educational Research (February, 1942), 35: 433-441.

deficiencies may result in a general up-grading of the work so that the group will be bunched more toward the top of the distribution on the end test, thus lowering the correlation between the readiness test and the end test scores.[1]

The reading demands on elementary algebra concern vocabulary and the reading of explanatory passages, symbolism, and problems. McKim[2] discovered that the correlation between achievement in algebra and the reading of explanatory passages was the highest of all correlations between that measure of achievement and any of the other separate reading tests.

Brownell[3] in his article on the evaluation of learning in arithmetic emphasizes the insights which are to be had from the continuous and enlightened questioning and observation of children while they are engaged in their daily work in arithmetic. Little research relating to arithmetic has been orientated with respect to the child and his problems in learning. Instruction and measurement cannot be kept apart in good practice.

1/L. J. Brueckner, "Development of Readiness Tests in Arithmetic," Journal of Educational Research (September, 1940), 34: 15-20.

2/M. G. McKim, The Reading of Verbal Material in Ninth Grade Algebra, Teachers College Contribution to Education, No. 850, Columbia University, New York, 1941, p.80.

3/W. A. Brownell, "Evaluation of Learning in Arithmetic," Arithmetic in General Education, Sixteenth Yearbook, National Council of Teachers of Mathematics, Bureau of Publications, Teachers College, Columbia University, New York, 1941, pp. 225-267.

Thompson[1] carried on an experiment with two groups of pupils for four years while they were going through grades VII through X. In the experimental group the average score in grade 10 was slightly above the 95th percentile according to the standard norms for public schools in the Columbia Research Bureau Algebra Test. Such exceptional and consistent results over a period of four years demonstrated that diagnostic and remedial individualized instruction is at least one very effective method of teaching mathematics.

In editing the Nineteen-forty Mental Measurements Yearbook,[2] Buros stated that the active participation of such a large number of competent reviewers in this test appraisal service marks the end of the days when authors and publishers could successfully market ill-conceived, poorly constructed, and inadequately validated tests with impunity. This yearbook grew from a 44-page non-critical bibliography in 1935 to a 700-page yearbook of critical reviews in 1941.

Gilmore[3] made a study of several cases needing

1/R. B. Thompson, "Diagnosis and Remedial Instruction in Mathematics," School Science and Mathematics (February 1941) 41: 125-128.

2/O. K. Buros, The Nineteen-forty Mental Measurements Yearbook, Mental Measurements Yearbook, Highland Park, New Jersey, 1941.

3/R. H. Gilmore, Corrective Arithmetic in Senior High School, Unpublished Master of Education Thesis, Boston University, School of Education, 1939.

Thompson[1] carried on an experiment with two groups of pupils for four years while they were going through grades VII through X. In the experimental group the average score in grade 10 was slightly above the 98th percentile according to the standard norms for public schools in the Columbia Research Bureau Algebra Test. Such exceptional and consistent results over a period of four years demonstrated that diagnosis and remedial individualized instruction is at least one very effective method of teaching mathematics.

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Gilmore[3] made a study of several cases needing

1\. E. Thompson. "Diagnosis and Remedial Instruction in Mathematics." School Science and Mathematics (February 1941) 41: 128-130.

2\. O. K. Buros. The Nineteen-Forty Mental Measurements Yearbook, Mental Measurements Yearbook. Highland Park, New Jersey, 1941.

3\. E. Gilmore. Corrective Arithmetic in Senior High School. Published Master of Education Thesis, Boston University School of Education, 1939.

corrective arithmetic in high school. Both Buckley[1] and Lund[2] investigated the deficiencies in arithmetic in better-than-average high schools. They found a real need for remedial instruction in the fundamental process in whole numbers through grade 12. Lund claims that in many cases these skills have been dulled by disuse and indifference.

A feeling of adequacy in arithmetic is produced by 100 percent accuracy. Such accuracy can be approached if not actually attained by corrective arithmetic through grade 12. This is one aspect of improving the attitude toward mathematics. We need to think carefully about the best methods, from the standpoint of psychology, of changing those attitudes that are unfavorable to the continued improvement of mathematical education.[3]

Much research has been done in the way of minute testing and drilling on fundamental processes in arithmetic, but very little has been done in analyzing and testing fundamental concepts of understanding number, number relationships, and

1/F. A. Buckley, What Need Is There for a Corrective Arithmetic Program in the Four Courses of a Senior High School?, Unpublished Master of Education Thesis, Boston University, School of Education, 1940.

2/R. J. Lund, How Much Lack of Mastery Is There in a Better-than-average High School in the Fundamental Skills of Arithmetic?, Unpublished Master of Education Thesis, Boston University, School of Education, 1943.

3/M. L. Hartung, "Attitudes Towards the Mathematics Curriculum and Post-war Planning," School Science and Mathematics (March, 1945),45: 273-278.

their expression. Such understanding is perhaps the basis of the greatest motivation for the effort required to master the fundamental skills. To work with numbers in abstract form, as in algebra, requires in elementary school a build-up in the meaning of number, number relationship, the expression of number relationship, through symbols, and the manipulation of such expressed values.

CHAPTER III

COMPARISONS BETWEEN SCORES ON STANDARD TESTS AND TEACHERS' MARKS

Introduction

In this chapter four tables and four corresponding charts are presented to show comparisons between individual scores on standard tests and teachers' marks in mathematics 9 and mathematics 10 for students who are new members of mathematics 11 classes. Mathematics 9 consists of one year of elementary algebra which is taken in grade 9, the last year of junior high school. Mathematics 10, taken in grade 10, the first year of senior high school, consists of a half year of both algebra and plane geometry integrated. The marks recorded for these courses are the letters A, B, C, and X, which are passing grades, and D and F which are failing grades. In the record files there is no differentiation among marks of a kind.

The three standard tests considered were all American Council on Education tests, namely: the Psychological Examination which was taken in grade 9, the Cooperative Elementary Algebra Test which was taken in grade 9, and the Cooperative General Mathematics Test which was taken in grade 10. The pupils of mathematics 11 classes were divided into five

groups according to their percentile ranks on each of the three standard tests mentioned. In each classification, group I includes those whose percentile ranks fall between 80 and 100; group II between 60 and 79; group III, between 40 and 59; group IV, between 20 and 39; and group V, between 0 and 19. Percentiles based upon the total population of the junior class and not upon the limited membership of mathematics 11 classes have been used because the degree of mental alertness is a factor of achievement in education.

Since passing work in mathematics 9 and mathematics 10 is a prerequisite for entrance into the mathematics 11 course, the pupils of mathematics 11 classes have no marks other than A, B, C, or X in the two preceeding mathematics courses. Whenever numerical averages were needed in working with these marks, A has been assigned the value 4; B, 3; C, 2; and X, 1.

It seemed to the writer that for practical purposes it would be permissible to assume that persons in any group I should make A in the courses under consideration; those in group II, B; those in group III, C; and those in group IV, X. The scores on the Psychological Examination are roughly indicative of the level of mental alertness of the student, but they should not be taken so seriously as to exclude other evidences of intelligence and talent in individual cases.[1/]

1/ L. L. Thurston and T. G. Thurston, Manual of Instructions, American Council on Education Psychological Examination for High School Students, American Council on Education, 744 Jackson Place, Washington, D.C., 1942.

Accepting this viewpoint, a pupil in group I is expected to make an A grade in mathematics, but if he earns a B grade, it is not considered alarming. If in the distribution of marks for group I, there are more B's than A's, the group is considered as rating below par. However, a person in group I who rates a C or an X in mathematics may be suspected of some maladjustment. There are three factors which determine achievement in mathematics. The Psychological Examination places the student according to his mental alertness. The teacher, in cooperation with the parents, can judge his industry. The third factor is his preparation, and diagnosis of his difficulties followed by remedial work is an attempt to overcome any existing deficiencies.

The distributing of marks in each of the other four groups is considered from this same point of view. Pupils whose marks in mathematics 9 or mathematics 10 place them in a group which is lower by two step intervals than the group in which their intelligence quotients from the Psychological Examination place them are considered maladjusted in some respect and in need of individual diagnosis.

Individual scores on the Cooperative Elementary Algebra Test have been compared with teachers' marks for mathematics 9 by using the five-group method already described.

Individual scores on the Cooperative General Mathematics Test have been compared with teachers' marks for mathematics 10

by using the five group method already described.

The four tables with accompanying charts and interpretations follow. In the tables, the numbers encircled represent those cases which are placed by teachers' marks at least two step intervals distant from the group in which their scores on standard tests place them.

There is also a table which summarizes the responses on the questionnaire concerning dislike for various branches of mathematics.

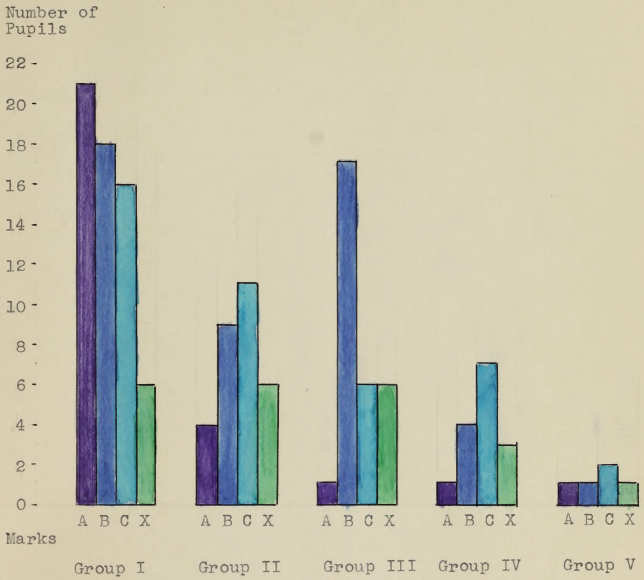
The distribution of marks earned in groups I, II, III, IV, and V on the American Council on Education Psychological Examination by 141 pupils in mathematics 9 shows a fairly even distribution of A's, B's, and C's in group I with 10 percent X's. The average for the group is only 2.9, but the mode is A. Group V shows a fairly even distribution of marks among A's, B's, C's, and X's. There are only five cases, but the chance of earning an A or B here is almost as good as for pupils in group I. The average is 2.4. The patterns for groups II and IV are similar to each other, and have a tendency to follow the normal curve. The average for group II is 2.4, lower than one would expect and for group IV 2.2, higher than expectancy. More than half the marks in group III are B's and the average is 2.4 which is a bit high. The averages for groups II, III, and V are all the same, namely 2.4. Only group I has the mode that one would anticipate.

Table 1. Distribution of Marks Earned in Groups I, II, III, IV, and V on the American Council on Education Psychological Examination by 141 Pupils of High School X in Mathematics 9.

Marks in Mathematics 9	Achievement Groups on Psychological Examination					
	I	II	III	IV	V	
	Number	Number	Number	Number	Number	Totals
A.....	21	4	①	①	①	28
B.....	18	9	17	④	①	49
C.....	⑯	11	6	7	2	42
X.....	⑥	⑥	6	3	1	22
Totals.....	61	30	30	15	5	141
Average....	2.9	2.4	2.4	2.2	2.4	
Mode.....	4	2	3	2	2	

For the purpose of computing numerical averages, A has been assigned the value 4; B, 3; C, 2; and X, 1.

The numbers encircled indicate the number of pupils whose marks in mathematics 9 differed by two step intervals from the group in which their scores on the Psychological Examination placed them.



Graph I. Distribution of Marks Earned in Groups I, II, III, IV, and V on the American Council on Education Psychological Examination by 141 Pupils of High School X in Mathematics 9.

Table 2. Distribution of Marks Earned in Groups I, II, III, IV, and V on the American Council on Education Psychological Examination by 142 Pupils of High School X in Mathematics 10.

Marks in Mathematics 10	Achievement Groups on Psychological Examination					
	I	II	III	IV	V	
	Number	Number	Number	Number	Number	Totals
A.....	16	2	①	0	0	19
B.....	21	6	8	⑥	②	43
C.....	⑱	18	16	5	2	59
X.....	⑥	④	6	4	1	21
Totals.....	61	30	31	15	5	142
Average.....	2.8	2.2	2.1	2.1	2.2	
Mode.....	3	2	2	3	2.5	

For the purpose of computing numerical averages, A has been assigned the value 4; B, 3; C, 2; and X, 1.

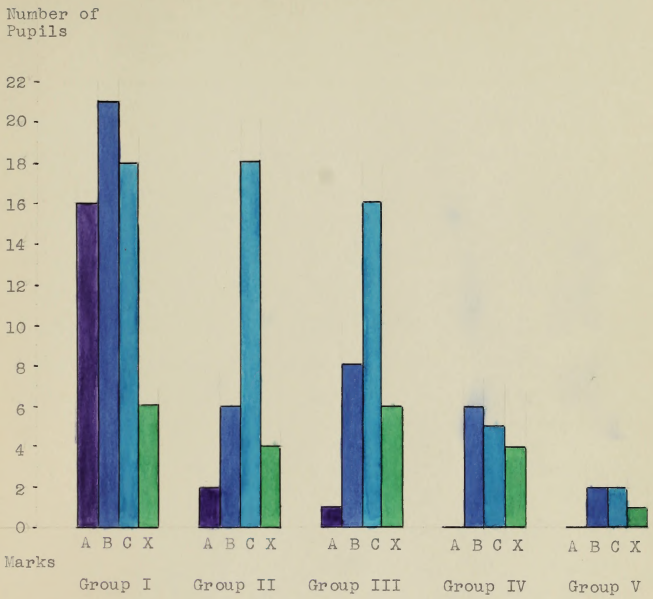
The numbers encircled indicate the number of pupils whose marks in mathematics 10 differed by two step intervals from the group in which their scores on the Psychological Examination placed them.

Table 2. Distribution of Marks Earned in Groups I, II, III, IV, and V on the American Council on Education Psychological Examination by 142 Pupils of High School X in Mathematics 10.

Marks in Mathematics 10	Achievement Groups on Psychological Examination				
	I	II	III	IV	V
	Number	Number	Number	Number	Number
A.....	16	8	(7)	0	10
B.....	21	6	8	(8)	(8) 43
C.....	(13)	18	16	5	2 80
X.....	(8)	(4)	6	4	1 21
Totals.....	61	30	37	18	5 152
Average.....	2.8	2.8	2.1	1.7	2.8
Mode.....	3	2	2	3	2.3

For the purpose of computing numerical averages, A has been assigned the value 4; B, 3; C, 2; and X, 1.

The numbers enclosed indicate the number of pupils whose marks in mathematics 10 differed by two step intervals from the group in which their scores on the Psychological Examination placed them.



Graph II. Distribution of Marks Earned in Groups I, II, III, IV, and V on American Council on Education Psychological Examination by 142 Pupils of High School X in Mathematics 10.

The distribution of marks earned in groups I, II, III, IV, and V on the American Council on Education Psychological Examination by 142 pupils in mathematics 10 shows a fairly even distribution of A's, B's and C's in Group I with 10 percent X's. The average, 2.8, is low for the group. In group II, 73 percent of the marks are below the expected B grade, and the average, 2.2, is low also. The distribution of marks in group III is normal with only one extreme case, an A. The average is 2.1. The patterns for groups IV and V are similar to each other, and the marks are high for both groups. The average for group IV is 2.1 and for group V 2.2. The average for group V is the same as for group II. Only group III has the mode that one would expect, namely, 2.

The distribution of marks earned in groups I, II, III, IV, and V on the American Council on Education Elementary Algebra Test by 142 pupils in mathematics 9 shows that 85 percent of the marks in group I are A's and B's which is good although more A's might be expected. The average for the group is 3.2, a bit low. Group II follows anticipation somewhat with 85 percent of the marks B's and C's. More B's and fewer C's would be a better distribution for this group. There are a few extreme cases, and the average is low at 2.5. Group III may be considered quite normal. However, there are two A's, extreme cases, and the average is 2.3. In group IV, 50 percent of the marks are X's and

Table 3. Distribution of Marks Earned in Groups I, II, III, IV, and V on the American Council on Education Cooperative Algebra Test by 142 Pupils of High School X in Mathematics 9.

Marks in Mathematics 9	Achievement Groups on Cooperative Elementary Algebra Test					
	I	II	III	IV	V	
	Number	Number	Number	Number	Number	Totals
A.....	19	2	②	0	②	25
B.....	21	15	6	②	④	48
C.....	⑥	13	13	8	7	47
X.....	①	③	3	11	4	22
Totals.....	47	33	24	21	17	142
Average.....	3.2	2.5	2.3	1.6	2.2	
Mode.....	3	3	2	1	2	

For the purpose of computing numerical averages, A has been assigned the value 4; B, 3; C, 2; and X, 1.

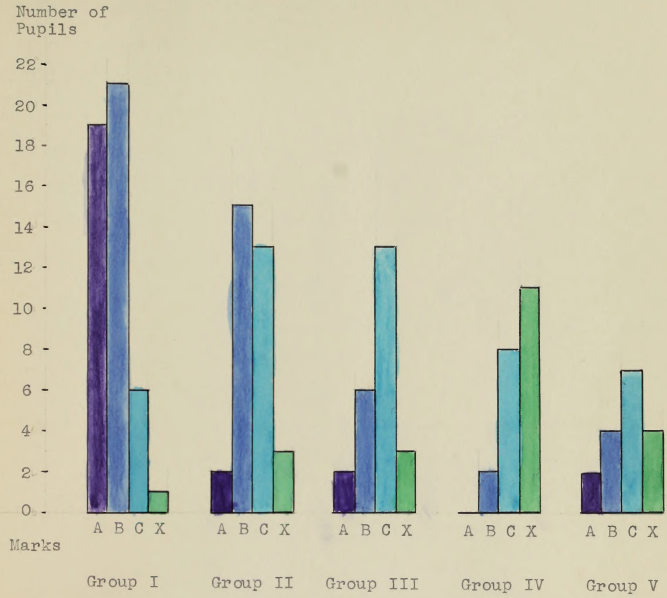
The numbers encircled indicate the number of pupils whose marks in mathematics 9 differed by two step intervals from the group in which their scores on the Cooperative Algebra Test placed them.

Table 3. Distribution of Marks Earned in Groups I, II, III, IV, and V on the American Council on Education Cooperative Algebra Test by 142 Pupils of High School X in Mathematics 9.

Marks in Mathematics 9	Achievement Groups on Cooperative Elementary Algebra Test				
	I	II	III	IV	V
	Number	Number	Number	Number	Number	Totals
A.....	10	2	②	0	①	25
B.....	21	15	6	③	④	49
C.....	⑤	15	15	8	4	47
X.....	①	③	3	11	4	22
Totals.....	47	35	24	21	14	142
Average.....	3.2	2.5	2.3	1.6	2.2	
Mode.....	3	3	2	1	2	

For the purpose of computing numerical averages, A has been assigned the value of 4; B, 3; C, 2; and X, 1.

The numbers encircled indicate the number of pupils whose marks in mathematics 9 differed by two or more intervals from the group in which their scores on the Cooperative Algebra Test placed them.



Graph III. Distribution of Marks Earned in Groups I, II, III, IV, and V on the American Council on Education Cooperative Algebra Test by 142 Pupils of High School X in Mathematics 9.

only 10 percent are extreme cases. The average is high at 1.6. The average for group V is very high, 2.2, and 35 percent of the marks are honor grades. The fact that the distribution of marks for groups I, II, III, and IV is so close to what one would expect is probably due to the fact that the Cooperative Elementary Algebra Test tests precisely the same kind of academic knowledge and skills that teachers require in first year algebra. Here the modes for groups II, III, and IV are the expected grades 3, 2, and 1 respectively.

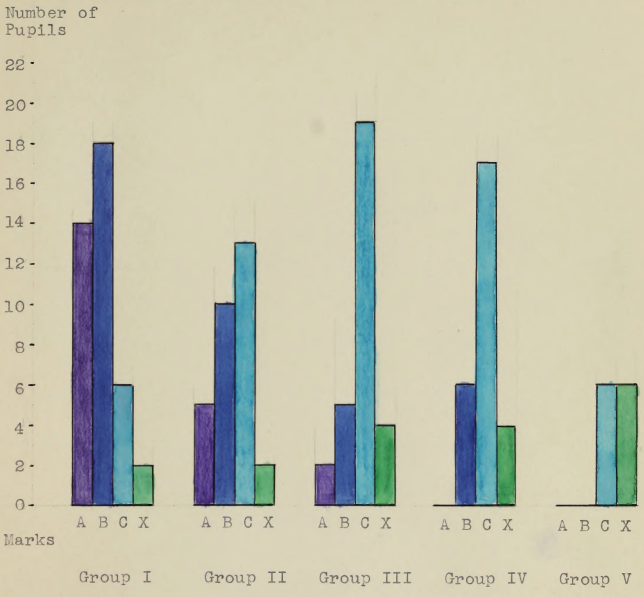
The distribution of marks earned in groups I, II, III, IV, and V on the American Council on Education General Mathematics Test by 139 pupils in mathematics 10 shows that 80 percent of the marks in group I are A's or B's and 20 percent of the marks are extreme cases. The average, 3.1 is low. In group II, 33 percent of the marks are B's and 60 percent are either A's or C's. The extreme cases make up the remaining 7 percent of the marks. The average is low at 2.6. In group III, 66 percent of the marks are C and only 7 percent of the marks are extremes, or A's. The average, 2.2 is about normal for the group. In group IV, 78 percent of the marks are C's and X's, and 22 percent of the marks are B's which is considered extreme in this group. The average is 2.1. In group V there are only C's and X's in equal numbers. The average is 1.5. The modes are the same for groups II, III, and IV.

Table 4. Distribution of Marks Earned in Groups I, II, III, IV, and V on the American Council on Education Cooperative General Mathematics Test by 139 Pupils of High School X in Mathematics 10.

Marks in Mathematics 10	Achievement Groups on Cooperative General Mathematics Test					
	I	II	III	IV	V	
	Number	Number	Number	Number	Number	Totals
A.....	14	5	②	0	0	21
B.....	18	10	5	⑥	0	39
C.....	⑥	13	19	17	6	61
X.....	②	②	4	4	6	18
Totals.....	40	30	30	27	12	139
Average.....	3.1	2.6	2.2	2.1	1.5	
Mode.....	3	2	2	2	1.5	

For the purpose of computing numerical averages, A has been assigned the value 4; B, 3; C, 2; and X, 1.

The numbers encircled indicate the number of pupils whose marks in mathematics 10 differed by two step intervals from the group in which their scores on the Cooperative General Mathematics Test placed them.



Graph IV. Distribution of Marks Earned in Groups I, II, III, IV, and V on the American Council on Education Cooperative General Mathematics Test by 139 Pupils of High School X in Mathematics 10.

Further General Interpretations

There seems to be a tendency for the marks in groups I and II to be lower than one would expect. Group III marks are normal in all tables except Table 1 where the marks run high. The marks in groups IV and V on all tables are quite high except for Table 4 where they might be considered just good for the group.

The arbitrary limits set for groups I, II, III, IV, and V and their associations with A's, B's, C's, and X's are not as precise as the writer would desire, but this was considered the best relationship that could be made since teachers' marks as recorded in the files do not form a continuous series. There is no differentiation of values for letters of a kind. The comparison used has been considered satisfactory to show significant trends. Furthermore, it is sufficient to indicate the pupils who fall far outside their group in mathematics achievement and who might be considered as cases suspicious of academic maladjustment.

It is interesting to note that the majority of the pupils in the two lowest groups on all tables earn much higher grades than their mental alertness scores would warrant, while those in the two highest groups represented on the tables receive grades lower than their mental alertness would deem probable. The middle group in most instances follows normal expectancy. It appears that the brighter

pupils are not achieving to their maximum capacity, and that the slower ones are achieving beyond their capacity.

The writer offers some interpretations of the possible causes. Persons in groups IV and V are apt to have very strong motives for studying mathematics. Mathematics may be a requirement of the institution which they hope to enter or for the vocations in which they have an intense interest. Educational systems are reluctant to have too many failures in a class, and as a result, teachers strive to eliminate failures. This situation may bring about undue focusing of attention on the encouragement and motivation of the slower pupils and on training them on the items the teacher considers most important and consequently weighs most heavily. These slower pupils, on the other hand, possess the industry and motivation to take advantage of this situation day by day, and thereby achieve good marks. They may be tutored and prepared for the type of questions that they will meet on class tests. They work with the teacher.

In all groups there are pupils who have the will to achieve to the best of their ability. In the two upper groups there are also pupils of high intelligence who will always pass their courses but who do not exert more effort than they see a need for. Their intelligience makes them utilitarian according to their own sense of future values. If they do not anticipate using mathematics, are not

preparing for college, and have not had any special interest for the subject aroused in them, they do not achieve to their maximum capacity. These pupils are often neglected by the busy teacher in her effort to fulfill what seems to be the more urgent duty to the slower pupils and to the college preparatory pupils. They are probably the most difficult type from whom to get maximum achievement because they need a very special kind of motivation. If a person receives a low B grade in mathematics, he is not apparently a case for concern, while actually he may be capable of superior work and may be wasteful of time and high ability.

There are always some who will not study because they are lazy or too occupied by other interests. There are, also, some who do not succeed in mathematics because they have not fully acquired some of the fundamental concepts of mathematics, and this lack interferes with the development and comprehension of subsequent mathematical ideas. It is the object of the case studies in the next chapter to investigate the nature of these concepts, for their development lies within the teachers' domain.

Summary of Questionnaire Responses

A questionnaire was given to 171 pupils of mathematics 11 classes in an attempt to discover their attitudes toward the various branches of mathematics which they had studied. They were asked to answer the questions frankly and honestly.

Of the 35 pupils who said that they disliked elementary arithmetic 25 expressed a dislike for one or more subsequent branches of mathematics also. There were 96 pupils who expressed a dislike for one or more branches of mathematics after completing the elementary arithmetic course which they did not dislike. Only three persons disliked the four branches considered on the questionnaire and fifteen disliked three branches. Thirteen of the pupils who claimed to like mathematics better than any other subject stated that they had disliked one or two of its branches. The largest uniform group consisted of 38 pupils who claimed to dislike only plane geometry. Further enumerations were as follows:

25 pupils liked mathematics better than any other subject.
40 pupils did not claim to like mathematics better than any other subject nor did they claim to dislike any branch of it.
35 pupils disliked arithmetic.
43 pupils disliked junior-high-school mathematics.
37 pupils disliked algebra.
69 pupils disliked geometry.
75 pupils disliked one branch of mathematics.
26 pupils disliked two branches of mathematics.
15 pupils disliked three branches of mathematics.
3 pupils disliked all four branches of mathematics mentioned.

It appeared that a pupil could like mathematics one year and dislike it the next or vice versa. A dislike for one branch of mathematics did not fix the attitude toward all of its branches nor toward the subject as a whole. For 70 percent of the pupils questioned the attitude toward mathematics

seemed to possess the characteristic of sporadic change. In individual cases, evidently, the feeling toward mathematics from time to time was determined by the pupil's current aims, his innate ability to cope with the difficulties of the subject, his environment, and his response to motivation from the teacher.

It was feared that pupils would be influenced by the repetition of the word "dislike" on the questionnaire, but the writer believes that the scatter of adverse attitudes does not substantiate this apprehension. A distribution of individual responses on the questionnaire has been presented in Table 5.

Table 5. Showing the Distribution of Individual Responses on the Questionnaire Given to 171 Members of Mathematics 11 Classes of High School X on November 20, 1945.

Number	Branch of Mathematics Disliked			
	Arithmetic	Junior H.S. Mathematics	Algebra	Geometry
	By Those Who Like Mathematics Best			
12				
4				x
4		x	x	
3	x			
1				
1	x	x		
	By Those Who Do Not Like Mathematics Best			
40				
38				x
14			x	
8		x		
7	x			
5			x	x
5		x		x
5	x	x		x
4	x	x		
4	x	x	x	
4	x	x		x
3		x	x	
3	x			x
3	x	x	x	x
2		x	x	x
1	x		x	
Totals	35	43	37	69

Table 8. Showing the Distribution of Individual Responses on the Questionnaire Given to 171 Members of Mathematics 11 Classes of High School X on November 20, 1946.

Number	Arithmetic	Junior H.S. Mathematics	Algebra	Geometry
	By Those Who Like Mathematics Best			
12				
4				x
6			x	
5		x		
1				
1		x		
	By Those Who Do Not Like Mathematics Best			
40				
38				x
14			x	
8		x		
7		x		
5			x	x
5		x		x
5	x	x		x
4	x	x		
4	x	x		
4	x	x		x
3		x	x	
3	x	x		x
3	x	x		x
2		x	x	x
1	x		x	
Totals	38	43	37	22

CHAPTER IV

CASE STUDIES

Introduction

In each of the six case studies presented in this chapter the writer has attempted to analyze the mathematical mistakes and attitudes of individuals, formed on the six standard tests and the questionnaire listed at the end of chapter I and collected in the appendix. This was done to diagnose certain basic difficulties that interfere with pupil progress in senior-high-school mathematics. The three all-mathematics tests, namely, the Iowa Every-Pupil Test of Basic Arithmetic Skills, the Cooperative Algebra Test, and the Cooperative General Mathematics Test were analyzed in their entirety. In the Psychological Examination the sections dealing with arithmetic problems and number series were analyzed. From the Mental Maturity Test, subtest 4, dealing with arithmetic problems and subtest 5, dealing with logic were analyzed. The subtests 2 and 6 covering spacial relationships and vocabulary, respectively, were evaluated. The results of the Occupation Interest Inventory and the responses on the questionnaire were weighed.

The writer has not aimed to diagnose deficiencies in

CHAPTER IV

CASE SAMPLES

Introduction

In each of the six case studies presented in this chapter the writer has attempted to analyze the mathematical mistakes and abilities of individuals, focused on the six standard tests and the questionnaire listed at the end of Chapter I and collected in the appendix. This was done to diagnose certain basic difficulties that interfere with pupil progress in senior-high-school mathematics. The three all-capable cases, namely, the Iowa Every-Pupil Test of Basic Arithmetic Skills, the Cooperative Algebra Test, and the Cooperative General Mathematics Test were analyzed in both cooperative and subjective manner. In the Psychological Examination the results dealing with arithmetic problems and number series were analyzed. From the Mental Maturity Test, subtest 4, dealing with arithmetic problems and subtest 5, dealing with logic were analyzed. The subtests 2 and 6 covering special relationships and vocabulary, respectively, were evaluated. The results of the Ocoupation Interest Inventory and the responses on the questionnaire were weighed.

The writer has not aimed to diagnose deficiencies in

the four fundamental processes with integers and fractions because research has shown that these deficiences exist in various degrees among high-school students and because there are approved ways to detect and correct these faults. The writer has tried to discover what concepts, understandings, and powers had not been developed sufficiently to afford satisfactory progress in senior-high-school mathematics.

In making each case study lists of the individual's mathematical mistakes appearing on the afore-mentioned standard tests in the chronological order in which they were given over a period of three years were compiled. Then the writer relied upon her insight, whatever had developed in twenty years of teaching boys and girls in mathematics classes, to fathom out and summarize the major sources of the difficulties that had persisted through the mathematics career of the individual.

Specific recommendations for individual cases require personal contact. Only records and performance on standard tests were available to the writer in making this study. Consequently, recommendations have not been made in each case but rather, the deficiencies found in all the case studies have been brought together in a summary and there some suggestions have been made as a means of eliminating these deficiencies.

Case No. 1

Case no. 1 was a boy, aged sixteen years and nine months, who is now a junior in High School X and a member of a mathematics 11 class. His local percentiles on standard tests and his marks in mathematics 9 and 10 were as follows:

	Percentile	Group
Psychological Examination	99	I
Cooperative Algebra Test	59	III
Cooperative General Mathematics Test	73	II

	Mark
Mathematics 9	X
Mathematics 10	X

In the Iowa Every-Pupil Basic Skills Test for Arithmetic taken in grade 8, he showed good judgment in estimating magnitudes that are used commonly, and no serious deficiencies in the fundamental processes. Errors made were of the type which showed a weakness in subject vocabulary and in the precise and implied meaning of words such as those used in discussing the properties of a fraction. Problems which involved several ideas of steps confused him. He seemed to lack concentration, focus, and the ability to think through a maze of ideas. His errors also showed the lack of ability to visualize a plan or diagram from a verbal description, or to visualize the completion of a diagram

Case No. 1

Case No. 1 was a boy, aged eighteen years and nine months, who is now a junior in High School X and a member of a mathematics 11 class. His local description on standard tests and his marks in mathematics 9 and 10 were as follows:

	Percentile	Group
Psychological Examination	99	I
Cooperative Algebra Test	80	III
Cooperative General Mathematics Test	75	II

Mark

Mathematics 9 X

Mathematics 10 X

In the Iowa Every-Pupil Basic Skills Test for Arithmetic Reasoning in grade 8, he showed good judgment in estimating magnitudes that are used commonly, and no serious deficiencies in the fundamental processes. Errors made were of the type which showed a weakness in subject vocabulary and in the precise and implied meaning of words such as those used in discussing the properties of a function. Problems which involved several ideas of steps confused him. He seemed to lack concentration, focus, and the ability to think through a maze of ideas. His errors also showed the lack of ability to visualize a plan or diagram from a verbal description, or to visualize the completion of a diagram

partly made. There were eight examples on the test not done, which might mean that he is a slow worker.

The Psychological Examination was taken after he had studied algebra for six months. Of the twenty problems in the arithmetic section, he failed three and omitted three The problems that he did not do correctly or omitted were long and involved, evidently containing too many ideas for him to comprehend. In the section on number series he did very well. The two wrong answers given involved the expression of a relationship involving subtraction.

The Cooperative Elementary Algebra Test was taken after nine months of studying algebra. On this test the weaknesses displayed on the eighth-grade arithmetic test were more noticeable and are listed here.

He was confused over the meaning of numerator and denominator and did not know what could be done with them.

He could not find the common denominator when combining fractions written in horizontal form although he did this correctly in arithmetic when fractions were written in vertical form.

He had difficulty transposing quantities in an equation when they were related through multiplication and division.

He was confused by multiple ideas in a problem.

He omitted examples which required the expressing of relationships between magnitudes either in equations or graphs.

He could not express a relationship between two values when subtraction was involved.

He was not clear on the meaning of symbolism for exponents and radicals.

He could not visualize a plan from a verbal description.

This test was not based on general intelligence used in everyday life but upon specific subject matter. The boy was in group III for this test and received X in mathematics 9 which is only a slight divergence.

The California Test of Mental Maturity was taken after five months in mathematics 10 where geometry is taught. In the arithmetic section of this test he had only one problem wrong out of fifteen, several of which involved space concept. There were four errors in fifteen questions on logic. Although the errors in logic concerned descriptions of spacial relationships, he exhibited much improvement in this respect since his study of geometry.

At the end of the sophomore year, the Cooperative General Mathematics Test was given. Here the weaknesses of the other two all-mathematics tests are again evident.

He lacked an understanding of the relationship between numerator and denominator and between dividend, divisor and quotient and their possible manipulations. He recognized this weakness and avoided problems when substituting fractions in a formula was required. He avoided literal fractional

equations. He was still weak in subject vocabulary and omitted examples in which complicated wording was involved. He could do a practical numerical problem and he could understand a practical numerical graph, but he did not comprehend the same relationships when they were expressed with symbols. During the mathematics 10 course, he acquired the understanding of ratio and gained in his recognition of spacial relationships. He did not retain the principles expressed verbally in theorems.

To summarize briefly, his difficulties come from three sources. There has been a persistent weakness in subject vocabulary and reading comprehension, especially in the reading of mathematics. This boy would be unable to profit by instructions from the average high-school textbook. There has been a weakness in the comprehension of spacial relationships. There has been lack of understanding of the use of symbols to express the relationship between values. He has a good sense of number values but he does not comprehend their expressed relationships. The expression of number relationships by means of symbols is the very essence of algebra. It is easy to understand why this boy stated on the questionnaire that he disliked algebra.

His percentile norms on the Occupational Interest Inventory based on the composite population were as follows:

equations. He was still weak in subject vocabulary and omitted exercises in which complicated wording was involved. He could do a practical numerical problem and be could understand a practical numerical graph, but he did not comprehend the same relationships when they were expressed with symbols. During the mathematics 30 course, he acquired the understanding of ratio and gained in his recognition of sound relationships. He did not retain the principles expressed verbally in theorems.

To summarize briefly, his difficulties come from three sources. There has been a persistent weakness in subject vocabulary and reading comprehension, especially in the reading of mathematics. This boy would be unable to profit by instruction from the average high-school textbook. There has been a weakness in the comprehension of special relationships. There has been lack of understanding of the use of symbols to express the relationship between values. He has a good sense of number values but he does not comprehend their expressed relationships. The expression of number relationships by means of symbols is the very essence of algebra. It is easy to understand why this boy stated on the questionnaire that he disliked algebra.

His percentile norms on the Georgemiouni Inventory based on the composite computation were as follows:

Fields of Interest

Personal-Social	70	Business	70
Natural	10	The Arts	80
Mechanical	40	The Sciences	10

Types of Interest

Verbal	80	Manipulative	60	Computational	40

Level of Interest 80

His position in the 10th percentile in the field of science indicates a lack of appreciation for underlying principles, the beginning and the growth of things, and fundamental relationships. This ties up with his lack of understanding of number relationships. His interest in the personal-social and in the business field and the fact that he is of the verbal type should provide helpful motivation toward his improvement in mathematical vocabulary and reading comprehension. His interest in art could be utilized to develop ideas of spacial relationships.

The weaknesses in certain fundamental concepts that have persisted through this boy's mathematical education have too often caused him not to be ready to comprehend sufficiently the new material presented. All this has probably contributed toward a restlessness given evidence by reports of absence, tardiness, and truancy; and the restlessness on the other hand, has impeded development.

Fields of Interest

Personal-Social	70	Business	70
Natural	10	The Arts	80
Mechanical	40	The Sciences	10

Types of Interest

Verbal	80	Manipulative	60	Computational	40

Level of Interest 80

His position in the IDEA percentile in the Field of science indicates a lack of appreciation for underlying principles. the beginning and the growth of things, and fundamental relationships. This ties up with his lack of understanding of number relationships. His interest in the personal-social and in the business field and the fact that he is of the verbal type should provide helpful motivation toward his improvement in mathematical vocabulary and reading comprehension. His interest in art could be utilized to develop ideas of spacial relationships.

The weaknesses in certain fundamental concepts that have persisted through this boy's mathematical education have too often caused him not to be ready to comprehend sufficiently the new material presented. All this has probably contributed toward a restlessness given evidence by reports of absence, tardiness, and truancy and the restlessness on the other hand, has impeded development.

Case No. 2

Case No. 2 was a boy aged sixteen years and one month, a junior in High School X and a member of a mathematics 11 class. His percentile ratings on standard tests and his marks in mathematics 9 and 10 were as follows:

	Percentile	Group
Psychological Examination	92	I
Cooperative Algebra Test	29	IV
Cooperative General Mathematics Test	94	I

	Mark
Mathematics 9	X
Mathematics 10	X

From the above data it can be seen that his achievement in mathematics 9 and 10 was much lower than his ranks on the Psychological Examination and on the General Mathematics Test would warrant. His score on the Iowa Every-Pupil Basic Skills Test for Arithmetic, taken at the end of grade 8, placed him in the 98th percentile in his class and at tenth grade level in arithmetic. There was no evident weakness disclosed on the section of this test which covered vocabulary and fundamental knowledge. On the remainder of the test difficulties with fractions occurred in four instances: two mistakes in expressing fraction values and two in multiplying mixed numbers. In a division example there were mistakes both in dividing and placing the decimal point. He did not find correctly the area

Case No. 2

Case No. 2 was a boy aged sixteen years and one month, a junior in High School X and a member of a mathematics II class. His percentile ratings on standard tests and his marks in mathematics 9 and 10 were as follows:

	Percentile Group	Mark
Psychological Examination	92	I
Cooperative Algebra Test	29	IV
Cooperative General Mathematics Test	94	I
Mathematics 9		X
Mathematics 10		X

From the above data it can be seen that his scholarship in mathematics 9 and 10 was much lower than his marks on the Psychological Examination and on the General Mathematics Test would warrant. His score on the Every-Pupil Basic Skills Test for Arithmetic, taken at the end of grade 8, placed him in the 88th percentile in his class and at best grade level in arithmetic. There was no evident weakness disclosed on the section of this test which covered vocabulary and fundamental knowledge. On the remainder of the test difficulties with fractions occurred in four instances: two mistakes in expressing fraction values and two in multiplying mixed numbers. In a division example there were mistakes both in dividing and placing the decimal point. He did not find correctly the area

of a triangle from facts given in a diagram. The last three examples were left incomplete.

On the arithmetic section of the Psychological Examination he had seven examples wrong out of twenty. Six of these involved fractions and one required the visualization of a diagram. He worked rapidly and accurately on the section dealing with number series. Although there were mistakes and those dealing with fractions and space concept were the principal types, this boy held very high rank in his class on both the tests just considered.

A sudden drop placed him in the 29th percentile in his class on the Cooperative Elementary Algebra Test at the end of grade 9. On this test the errors in fractions were still numerous and a lack of space concept was still evident. He could not work with signed numbers, could not express relationships by using symbolism, and did not know how to manipulate the terms of an equation. These topics very definitely belong to a first-year algebra course.

On the Mental Maturity Test he was in the 86th percentile of his class. On the problem section there were two mistakes which involved fractions, two which involved percents, and one which concerned both fractions and percents. On the logic section of this test he made three wrong deductions out of fifteen, and two of these errors were due to assuming that the converse of a statement is always true. On the

section dealing with the manipulation of spacial relationships he had only one correct answer out of a possible fifteen. This indicates serious weakness in space concept.

His score on the Cooperative General Mathematics Test taken in grade 10 placed him in the 94th percentile of his class. The errors on this test involved fractions, space concept, relationships expressed through symbols and graphs, and the manipulating of terms in an equation. There were other mistakes falling outside the scope of a high-school sophomore class.

His percentile norms based on the composite population for the Occupational Interest Inventory were as follows:

Fields of Interest

Personal-Social	40	Business	70
Natural	70	The Arts	50
Mechanical	70	The Sciences	10

Types of Interest

Verbal	60	Manipulative	20	Computational	30

Level of Interest 70

His low ratings in science and in computational type of work showed a great lack of interest in the kind of thinking that is done in mathematics. This boy did good work in arithmetic through grade 8 although he did not attain marks equal to his 98th percentile rating on the standard arithmetic test. The writer thinks that a habit of nonexertion might have

developed which was incompatible with the exacting task of making the necessary generalizations in algebra. The standard algebra test checked the knowledge acquired in the algebra class and he had not put forth enough effort. Lack of space concept was another factor which contributed toward his poor work in mathematics 10. The records described him as a pleasant likeable boy. Because of these traits he may have avoided a check upon his inert attitude toward mathematics. Poor study habits were probably the chief difficulties that interfered with his progress. He has aroused himself and has done good work in mathematics 11.

Case No. 3

Case no. 3 was a girl in mathematics 11, aged fifteen years and ten months, and a junior in High School X. On the questionnaire, she claimed that she disliked elementary-school arithmetic, junior-high-school mathematics, and algebra.

Her ranks on three standard tests and her marks in mathematics 9 and 10 were as follows:

	Percentile	Group
Psychological Examination	84	I
Cooperative Algebra Test	62	II
Cooperative General Mathematics Test	60	II

		Mark
Mathematics	9	X
Mathematics	10	C

developed which was incompatible with the existing track of
making the necessary generalizations in algebra. The stand-
ard algebra test checked the knowledge acquired in the algebra
class and he had not put forth enough effort. Lack of space
caused was another factor which contributed toward his poor
work in mathematics 10. The records described him as a disap-
ant likeable boy. Because of these traits he may have avoided
a check upon his inept attitude toward mathematics. Poor study
habits were probably the chief difficulties that interfered
with his progress. He has matured himself and has done good
work in mathematics 11.

Case No. 3

Case no. 3 was a girl in mathematics 11, aged fifteen
years and ten months, and a junior in High School X. On the
questionnaire, she claimed that she disliked elementary-school
arithmetic, junior-high-school mathematics, and algebra.

Her marks on three standard tests and her marks in mathe-
matics 9 and 10 were as follows:

	Mark	Percentile Group
Psychological Examination	84	I
Cooperative Algebra Test	82	II
Cooperative General Mathematics Test	80	II

	Mark
Mathematics 9	X
Mathematics 10	C

On the section dealing with vocabulary and fundamental knowledge of the Iowa Every-Pupil Basic Skills Test for Arithmetic, taken in grade 8, she made four mistakes due to not knowing the meaning and value of fractions and five mistakes concerning the meaning of words and expressions pertaining to mathematics. There were three mistakes which involved the comparison of values, namely: large whole numbers, decimals, and fractions. She did not recognize the value of 66 x 100 ÷ 4 as being the same as 66 multiplied by 25. She did not know the meaning of the position of a digit in a number, made mistakes in multiplication and division when zero was involved, and did not recognize an application of the circle formula. There was evidence of lapse of memory or confusion in that she forgot to carry one in addition or forgot that one had been borrowed in subtraction. In one example requiring the subtraction of mixed numbers, she subtracted the fractions correctly and then forgot she was subtracting and added the whole numbers. In three instances denominate number exercises were wrong because she did not follow through by changing a fraction of a foot to inches or a fraction of a minute to seconds. There were two mistakes involving the meaning of percents, one each in adding and multiplying mixed numbers, and one where spacial visualization was involved. Six of her answers were so different from the correct results that a lack of number sense was quite

evident. For a person lacking this number sense there is little meaning in the expression of number values, and even less meaning in the manipulation of such expressed values. In the case of this girl, the result was confusion. At the end of grade eight she had not acquired the power necessary for the study of algebra, and furthermore, she was aware of and perturbed by its absence.

On the problem section of the Psychology Examination, three problems involving number comparisons and three involving fractions were incorrect. She answered only one-third of the questions on number series and one fourth of those completed were wrong.

This shows a slowness in recognizing number relationships.

On the Elementary Algebra Test there were six errors in signs, two of which involved parentheses. The two graph questions which she attempted to do were wrong. There were six errors in fractions, two of these having the wrong common denominator. There were eight errors in expressing relationships either through simple multiplication and division with their symbolisms, spacial concepts, tables, or numerical comparisons. There were errors in solving a formula after substitution, and in solving a very simple radical equation.

On the Mental Maturity Test, problem section, seven examples out of a total of fifteen were wrong. Four of these

involved comparisons of sets of numbers, two concerned space concept, and one dealt with percentage relationship. On the logic section, her performance in making thirteen correct deductions out of a possible fifteen indicates a rather logical mind. On the section dealing with difficult manipulation of spacial relationships, she had five right responses out of a possible fifteen. This shows a degree of weakness in space concept.

On the Cooperative General Mathematics Test there were three sign mistakes, four errors or chosen omissions involving the comparison of numbers, two wrong ideas in the meaning of complement, and four errors in geometric facts. There were two mistakes in literal fractional equations, two where a knowledge of the relationship between numerator and denominator was necessary, and one in each of the following: multiplying radical expressions, use of a simultaneous equation graph, percentage, and in an incomplete check which resulted in not detecting an error. Noncomprehension of the relationship between numerator and denominator of a fraction was evident on each of her standard mathematics tests: arithmetic, algebra, and general mathematics. This is the source of her difficulties with fractions and fractional equations.

This girl has not seen the logical development of number because she could not comprehend number itself from the very beginning of her formal education. Consequently, she has

tried to memorize the steps in various processes without seeing them as links that hold together in a chain. It appears that she has tried to follow direction without reasoning and without a purpose, but, in the midst of it her mental capacity for logic has asked, "Why?" This has thrown her off the course and confusion has resulted. Repeating occurences of this sort have caused frustration which she expressed in her dislike for mathematics through grade 9. She has not expressed a dislike for plane geometry, probably because it deals with two dimensional figures which have meaning for her, and because its direct appeal to logic has reached her.

Her percentile norms on the Occupational Interest Inventory based on the composite population were as follows:

Fields of Interest

Personal-Social	90	Business	80
Natural	10	The Arts	90
Mechanical	40	The Sciences	1

Types of Interest

Verbal	90	Manipulative	60	Computational	50

Level of Interest 80

Her very low rank in the field of science--which includes activities related to research, experimentation, invention, the determination of cause and effect relationships, and controlled observation--indicates a lack of native interest in

the mental activities employed in the study of mathematics. Perhaps she has learned as well as can be expected in relation to her innate ability. If this is not the case, then an appeal to her strong art interest might have been utilized to develop the sense of spacial relationships and this could be integrated with number values. Her interest in the personal-social field and in the business field also could be utilized to develop a number sense. The next step would be the logical building-up of number relationships. These things might have been done in the beginning to prepare her for the study of mathematics, or at anytime thereafter as remedial measures. If such appeals reached her, they would tend to remove the dislike and frustration caused by the subject and thus improve achievement. If such appeals have been made without success, then one must conclude that she has learned as well as can be expected in relation to her innate ability.

Case No. 4

Case no. 4 was a boy, aged sixteen years and one month, who is a junior in High School X and a member of a mathematics 11 class. His local percentile ranks on standard tests and his marks for mathematics 9 and 10 were as follows:

	Percentile	Group
Psychological Examination	87	I
Cooperative Algebra Test	55	III
Cooperative General Mathematics Test	90	I

	Mark
Mathematics 9	C
Mathematics 10	C

An analysis of the mistakes made in the section dealing with vocabulary and fundamental knowledge on the Iowa Every-Pupil Basic Skills Test for Arithmetic disclosed the following weaknesses. There were four mistakes in estimating measurements and four in subject vocabulary. He did not recognize the relationship between numerator and denonimator, an application of the circle formula, or a diagram that illustrated a fraction value. He did not comprehend the position value of a digit in a number or the value of a number expressed in terms of other numbers and symbols. There were deficiencies in the fundamental processes: four errors in the multiplication of integers and four in working with mixed numbers. There were three mistakes which involved the

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Case No. 4

Case no. 4 was a boy, aged sixteen years and one month, who is a junior in High School X and a member of a mathematics 11 class. His local percentile ranks on standard tests and like marks for mathematics 9 and 10 were as follows:

Percentile Group		
Psychological Examination	87	I
Cooperative Algebra Test	56	III
Cooperative General Mathematics Test	80	I

Mark	
Mathematics 9	C
Mathematics 10	C

An analysis of the mistakes made in the section dealing with vocabulary and fundamental knowledge on the Iowa Every-Pupil Basic Skills Test for Arithmetic disclosed the following weaknesses. There were four mistakes in estimating measurements and four in symbols vocabulary. He did not recognize the relationship between numerator and denominator, an application of the circle formula, or a diagram that illustrated a fraction value. He did not comprehend the position value of a digit in a number or the value of a number expressed in terms of other numbers and symbols. There were deficiencies in the fundamental processes: four errors in the multiplication of integers and four in working with mixed numbers. There were three mistakes which involved the

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visualization of spacial relationships, and two which required the addition of items selected from a paragraph. Two answers were so different from the correct answers as to be lacking in common sense and a simple comparison between denominate numbers was not observed.

On the arithmetic problem section of the Psychological Examination taken in grade 9, he had twelve examples wrong out of twenty. Eight of these contained fractions, four involved the comparison of numerical values, and in one question he did not read all the facts. All the problems that he missed contained many facts stated in rapid sequence. It appeared that the complexity of ideas was too much for him to comprehend.

In the section on number series he made a good record. He was able to see number relationships when set to it directly.

On the Cooperative Elementary Algebra Test taken in grade 9, there were three mistakes in signs and eight mistakes in expressing a value in terms of other values by using symbolism. In several instances his answer was the inversion of the correct result. In another case he inverted a fraction when either reading or writing it. All questions dealing with graphs were omitted: two at the beginning of the section and four at the end. There seems to be a deficiency in perceiving spacial relationships as well as abstract relationships. There were also three mistakes in reducing fractions and five in

solving fractional equations. The word sum was not comprehended, and mistakes were made in multiplying and subtracting monomials.

On the arithmetic problem section of the Mental Maturity Test taken in grade 10, he missed three examples which required the use of space concept. There were mistakes in three other problems which he was unable to comprehend because of the multiplicity and congestion of facts occurring in them.

He did well on the logic section by answering thirteen out of fifteen questions correctly. On the section dealing with difficult manipulations of spacial relationships he made a poor score--six right out of the fifteen, that he attempted. His score on the vocabulary test was low also.

On the Cooperative General Mathematics Test taken in grade 10, three errors involving space concept occurred. There were five mistakes in examples where the relationship between values was given through ratios or numerical comparisons. Two questions which required the equating of formulas were answered incorrectly. All questions or graphs were omitted except a simple temperature graph. He could not answer the question concerning the relationship between divisor, dividend and quotient.

This boy's chief difficulty seems to be poor reading comprehension and inadequate subject vocabulary. He does not always get the right idea from the printed page and he is

weak in the association of ideas that should cohere. This was noticeable on tests given in the three successive years, grades 8 through 10. He is weak in spacial concept and especially deficient in visualizing spacial relations from verbal descriptions. His number sense needs developing and he needs practice to acquire skill in the fundamental processes. All these weaknesses were apparent in successive tests. Because he does not comprehend these relationships, he has little success in expressing them in algebraic symbols.

His percentile norms on the Occupational Interest Inventory based on the composite population were as follows:

Fields of Interest

Personal-Social	20	Business	30
Natural	90	The Arts	50
Mechanical	60	The Sciences	50

Types of Interest

Verbal	30	Manipulative	40	Computational	30

Level of Interest 80

His very low rating in the personal-social field and in verbal interest are closely connected with his weakness in verbal and reading comprehension of mathematical ideas. The low scores in the field of business and computational interests coincide with his poor achievement with the fundamental processes in arithmetic and his lack of number sense. His high score in the natural field suggest an area for individual

motivation. His middle-rate scores in the arts and sciences offer hope of success in visualizing spacial relationships and perceiving abstract relationships through which the verbal or reading comprehension should be improved.

On the questionnaire he said that he disliked arithmetic but that he likes mathematics better than any other subject. The records described him as an introvert, a shy boy who does not like to enter into class discussions. The fact that convention does not require each individual in a mathematics class to take part in a discussion may be his reason for liking mathematics better than any other subject.

Case No. 5

Case no. 5 was a boy, aged fifteen years and six months, a junior in High School X, and a member of a mathematics 11 class. His percentile ratings on standard tests and his marks in mathematics 9 and 10 were as follows:

	Percentile	Group
Psychological Examination	81	I
Cooperative Algebra Test	80	I
Cooperative General Mathematics Test	57	III

	Mark
Mathematics 9	C
Mathematics 10	C

This boy entered the school system, of which High School X is a unit, in grade 9. Consequently, he did not take the

Iowa Every-Pupil Basic Skills Test for Arithmetic which was administered in grade 8. The first standard test available in the diagnosis of his difficulties was the Psychological Examination which he took in grade 9. On the problem section of this test he computed twelve examples correctly out of a possible twenty. He omitted one example which required the forming and equating of two ratios and the solution of the resulting proportion. In the space allowed for scribbling he wrote out every example, although most of them involved numbers which should have been manipulated mentally. Consequently, he did not have time to do the last seven problems.

On the Cooperative Algebra Test taken in grade 9, the outstanding weakness of this boy was his inability to express relationships between literal values. Six instances occurred where he could not express literal relationships in subtraction, multiplication, division, proportion, percentage, and in a combination of subtraction and multiplication. He did incorrectly one area problem and omitted another, both of which were explicit and involved only the expression of spacial relationships. There was one question for which he did not recognize the equation which matched the problem. Another weakness was his lack of understanding of fractions. There were three instances in which he could not reduce algebraic fractions ranging from the simple form $\frac{r}{r}$ which he said equalled o to fractions of mediocre difficulty

requiring division by monomial and binomial factors. An example in the subtraction of fractions was omitted, and two mistakes occurred in literal fractional equations. He omitted two problems based upon the direct comparison of numbers. Three errors were due to inability to recognize the meaning and significance of exponents and to manipulate them. One obvious error in sign and one in removing parentheses occurred. He carelessly read a bar graph wrong and omitted one theory question on graphing within the range of first year algebra.

On the problem section of the Mental Maturity Test taken in grade 10 he had ten problems right out of fifteen. Two mistakes involved the expression of relationships through numbers; one by multiplication and one by subtraction. There was one mistake in an example requiring space concept and one requiring a comparison of fractional parts. He omitted the last example, one on ratio. Two of his answers showed lack of common sense in respect to an approximate answer.

On the section dealing with spacial relationships he did very poorly. He did very well on the logic test and about average on the vocabulary test.

On the Cooperative General Mathematics Test taken in grade 10, four errors were due to inadequate subject vocabulary and two mistakes were caused by missing the significance of one word or idea in reading the problem. There were four mistakes involving subtraction: two in subtracting signed

numbers and two in expressing relationships by subtraction. Four mistakes and one omission evidenced lack of ability to perceive spacial relationships. Three problems using implicit facts were wrong, and three graph questions were omitted. There were two mistakes in working with literal fractional equations and two in simplifying expressions after substitutions had been made.

A summary of the most persistent deficiencies in this boy's mathematics from grade 9 to grade 11 would include his inability to express number relationships, his lack of understanding of fractions, and his lack of space concept. His inability to express relationships by using the fundamental processes, especially subtraction, was noticeable. His weak subject vocabulary and poor reading comprehension of mathematics showed up directly in a few spots but may be an underlying factor in other deficiencies.

His percentile norms for the composite population of the Occupational Interest Inventory were as follows:

Fields of Interest

Personal-Social	30	Business	80
Natural	10	The Arts	20
Mechanical	90	The Sciences	50

Types of Interest

Verbal	20	Manipulative	90	Computational	70

Level of Interest 70

His above-average score in computational interest and his average rating in the field of science suggest innate capacity, for slightly above-average work in mathematics. His low rank in the arts shows a lack of appreciation of line values and spacial relationships. His low rank in verbal interest was apparent through inadequate subject vocabulary and poor mathematical reading comprehension. This deficiency also interferes with a pupil's expression of relationships by obstructing the formation of the idea. It also impedes understandings which come through verbal explanations.

On the questionnaire he claimed to dislike arithmetic, junior-high-school mathematics, and geometry. With his computational interest capacity a liking for arithmetic would be expected. A lack of understanding because of verbal deficiencies and a mechanical manner of working may have contributed toward this dislike. His deficiency in perceiving spacial relationships and his weakness in working problems containing implicit facts were factors in the dislike for geometry; and all these items entered into his dislike for junior-high-school mathematics.

It should be noted that his position in the 80th and 81st percentiles for two of the standard tests barely placed him in group I for each test. The numerical value of C was not given in the records but from the marking period marks one would judge that his C's were not low. Consequently, the

divergence between his marks in mathematics 9 and 10 and his scores on standard tests is not as great as a first glance implies.

The deficiencies in his mathematics foundation have caused him to become less able to progress as the work of the subject advanced. He failed the first term of mathematics 11.

Case No. 6

Case no. 6 was a boy, aged sixteen years and ten months, a senior in High School X, and a member of a mathematics 11 class. His percentile ratings on standard tests and his marks in mathematics 9 and 10 were as follows:

	Percentile	Group
Psychological Examination	92	I
Cooperative Algebra Test	73	II
Cooperative General Mathematics Test	87	I

		Mark
Mathematics	9	C
Mathematics	10	X

This boy was a repeater in mathematics 11 because he failed the course the first time that he took it. His first standard test available for analysis was the Psychological Examination taken in grade 10. On the problem section five mistakes occurred in problems in which it was necessary to express a relationship between whole numbers in a ratio less than one. Two mistakes resulted from not using all the facts

stated in the problems: the answers given were not the complete answers. Other mistakes involved a comparison of fraction values, working with space concept, and simple addition of integers. The problems that he did correctly were practical problems of everyday life stated in commonplace language. Those that he failed, besides containing the mathematical ideas mentioned, were of a less practical type and some of them were stated in English expressions, the meaning of which required some thought.

He did exceptionally well on the section dealing with number series by completing a goodly number within the time limit with only one mistake.

On the problem section of the Mental Maturity Test taken in grade 11 six solutions were wrong out a total of 15. Again, four errors occurred in problems involving number comparisons: two dealing with whole numbers, one with fractions, and one with decimals. As before, the wording of these problems was a bit complicated. The other two mistakes concerned the expressing of spacial relationships with no implicit facts involved.

His logic test was excellent: only one wrong deduction out of 15. His score on the spacial relationship test was very high, and his vocabulary rating was good.

On the Cooperative General Mathematics Test taken in grade 11, two equation problems showed the lack of ability

to transpose quantities by means of multiplication and division, and two problems requiring the use of space concept were wrong. Applications of the precise meaning of the word, complement, were missed twice and the correct relationship between dividend, divisor, and quotient was not utilized. He omitted two graph questions and answered one incorrectly. One mistake occurred in a problem requiring an expressed relationship between two fractions when the wording was a bit complicated. A decimal point was misplaced, and a sign was wrong in subtraction.

This boy's work evidenced serious lack of ability to compare or relate whole numbers one to another especially where the relationship between the whole numbers resulted in a ratio less than one. In spots he showed the ability to perceive spacial relationships but fell down when it came to expressing them. He seemed not to have developed the fraction concept, so he made many mistakes in expressing fraction relationships and in working with fraction forms. Problems containing many ideas intricately stated seemed to evade him. He either did not know how or did not bother to take problems apart bit by bit in order to comprehend exactly the conditions described in the problem. He was not accurate as single errors of various types were found scattered throughout the tests.

His percentile norms on the Occupational Interest Inventory based on the composite population were as follows:

Fields of Interest

Personal-Social	10	Business	60
Natural	90	The Arts	50
Mechanical	50	The Sciences	60

Types of Interest

Verbal	50	Manipulative	10	Computational	70

Level of Interest 90

On the Occupational Interest Inventory his highest percentile, 90, appeared for level of interest. This showed a strong desire to engage in activities above those of a simple, unskilled, routine nature. His next highest rating was a percentile of 80 in the natural field of interest. There was nothing in his record that seemed to tie up with this interest. His work in biology was poor, X for both terms, and his choices at various times for a school of higher education were a technical school, a law school, a liberal arts college, and finally a school of business administration. The writer wonders what significance there is in his interest in the natural field and if anything might have been accomplished through it. The low ratings in the personal-social field and in manipulative interest ought not to interfere with his achievement in mathematics. The remaining types and fields of interest which might affect achievement in mathematics were average or above, thus indicating that he has no innate interest drawbacks.

The records intimate that he is a boy who does not always apply himself to the lesson at hand and who frequently misbehaves, though not maliciously. It appeared to the writer that as he pursued mathematics, he never developed adequately the fraction concept and an understanding of how to express relationships between quantities. These deficiencies seemed to be the source of most of his difficulties. The study of algebra requires a high development of these ideas so it was not surprising to find that on the questionnaire he claimed a dislike for algebra.

Summary

The six cases considered were pupils in mathematics 11 classes in a better-than-average high school. The percentile ratings for pupils in this school, on the many standard tests administered, were considerably higher than standard norms. The six pupils whose tests were analyzed earned good scores on these standard tests but their achievement in mathematics classes did not parallel their measured intelligence. The idea of this study was not to disparage the good teaching done in this excellent school system but rather to analyze some cases which fall outside the margin of satisfactory progress, for the purpose of discovering the difficulties that interfere with satisfactory progress in senior-high-school mathematics. The plan resembled the procedure that takes place continually in industry as a means of eliminating

flaws and getting better production.

From this point of view the writer offers a summary of the weaknesses in concepts and understandings that were discovered. Most outstanding were the lack of ability to express relationships, lack of the fraction concept, and lack of development of space concept. There were many mistakes in expressing relationships between quantities by using the fundamental processes of subtraction, multiplication, and division. The inability to use subtraction was surprising. It seemed that the fact that if $a + b = c$, then it is true also that $c - a = b$ and $c - b = a$ must be brought to pupils' attention. Some pupils do not make this inference or generalization without the help of the teacher although it seems obvious to anyone who recognizes it. These pupils should explain in words what they are doing in the fundamental processes and should have more instruction and practice in using the fundamental processes to express relationships between numbers. This basic ability is necessary before more complicated relationships of a general nature can be expressed meaningfully. The many mistakes made in the meaning of a fraction, expressing fractions, and working with fractions showed a lack of the fraction concept. Fractions were worked mechanically without understanding. Thiele[1] says, "Teachers are not to be

1/C. L. Thiele, "The Mathematical Viewpoint Applied to the Teaching of Elementary School Arithmetic," Chap. XI, Teaching Arithmetic, Tenth Yearbook, National Council of Teachers of Mathematics, Bureau of Publications, Teachers College, Columbia University, New York, 1935, p. 225.

flaws and pointing further investigation.

From this point of view the writer offers a summary of the weaknesses in concepts and understandings that were discovered. Most outstanding were the lack of ability to express relationships, lack of the fraction concept, and lack of development of those concepts. There were many mistakes in expressing relationships between quantities by making the fundamental processes of subtraction, multiplication, and division. The inability to use subtraction was surprising. It seemed that the fact that if $a + b = c$, then it is true also that $c - a = b$ and $c - b = a$ must be brought to pupils' attention. Some pupils do not make this inference or generalization without the help of the teacher although it seems obvious to anyone who recognizes it. These pupils should explain in words what they are doing in the fundamental processes and should have more instruction and practice in using the fundamental processes to express relationships between numbers. This basic ability is necessary before more complicated relationship or a general nature can be expressed meaningfully. The many mistakes made in the solving of a problem, expressing fractions, and working with fractions showed a lack of the fraction concept. Fractions were worked mechanically without understanding. Tuttle says, "Teachers are not to be

. Buffalo, "The Mathematical Viewpoint applied to the Teaching of Elementary School Arithmetic," Chap. XI, Teaching Arithmetic. Forty-Fourth Yearbook, National Council of Teachers of Mathematics. New York, 1945, p. 324.

censored too severely for this if the space devoted in the textbook to the development of the fraction concept is cut to a minimum so as to provide room in the textbook for the many specific difficulties into which the topic has been analyzed." Wheat[1] makes the following statement:

> The history of fractions reveals that the race made little if any progress in development of ideas of fractions so long as size was emphasized at the expense of number, or so long as number was emphasized at the expense of size. The same revelation is to be noted in the case histories of pupils in school. They make no progress in developing a working understanding of fractions so long as either distinguishing feature of the fraction is neglected in their thinking. Pupils must be led to study size as well as number.

The lack of space concept seemed to be due to lack of development rather than to the lack of innate ability. The use of more diagrams, simple blue print drawings, and graphs in arithmetic would help. McLelland and Dewey[2] stated, "That the Greeks also kept their arithmetical training in closest connection with the study of spacial forms, with measurement, may again be more than a coincidence." In junior high school a course in mechanical drawing integrated with arithmetic could be utilized to further develop spacial relationships along with

1/H. G. Wheat, "A Theory of Instruction for the Middle Grades," Chap. V, Arithmetic in General Education, Sixteenth Yearbook, National Council of Teachers of Mathematics, Bureau of Publications, Teachers College, Columbia University, New York, 1941, pp. 99-100.

2/J. A. McLellan and J. Dewey, Psychology of Number, D. Appleton and Company, New York, 1895, p. xiii.

number sense, subject vocabulary, and terminology. Next on the list of difficulties according to frequency were a lack of number sense and poor reading comprehension of mathematics. These difficulties appeared at an earlier stage in the child's formal education than the group first mentioned. The lack of number sense was apparent in pupils who could not do the following things: estimate the lengths of common objects, make approximate comparisons between large numbers, sense an approximate answer as a check on a careless mistake, recognize the comparative magnitudes of common fractions and decimals, and recognize the position value of a digit in a number. Brownell[1] says,

> Quantitative thinking is probably the most complicated thing children face in elementary school. Number is abstract. There is no special arithmetic instinct. Nature does not provide any tangible evidence of number. A person must have the thought pattern '5' before he can impose the number '5' on objects....[2] Primary number is taught as if skills acquired mechanically would later surely take meaning, and verbalizations memorized unintelligently would later inevitably become well-rounded concepts....Children must from the start see arithmetic as an intelligible system if they are ever to be intelligent in arithmetic.

One case study showed very plainly that the lack of number sense resulted in confusion and frustration throughout the study of mathematics.

1/ W. A. Brownell, "Psychological Considerations in the Learning and Teaching of Arithmetic," Chap. I, Teaching of Arithmetic, Tenth Yearbook, National Council of Teachers of Mathematics, Bureau of Publications, Teachers College, Columbia University, New York, 1935, p. 20.

2/ Ibid, p. 31.

Poor mathematical reading comprehension was noticeable in problems when an idea escaped the pupil and in intricately expressed problems which the pupil could not comprehend. Weakness in subject vocabulary was another phase. In some cases there was an innate verbal weakness and in others there was evidently a lack in focus of attention. Lack of number sense and poor mathematical reading comprehension are weaknesses which should be detected and treated as individual matters for correction.

There were two cases in which lack of sustained attention and poor study habits seemed to be factors of poor progress. In one instance these were connected with lack of understanding, but in the other only the habits were responsible.

Weaknesses in concepts and understandings that have been mentioned persisted as the individuals considered pursued mathematics courses. Because of these deficiencies the individuals were unready and unable to build up new mathematical ideas. In each case certain spots in the mathematical foundstions were weak, and any structure attempted above these weak spots crumbled.

The writer thinks that more emphasis should be placed on meanings and understandings in the teaching of arithmetic and mathematics. As a check diagnosis of difficulties resulting from the lack of fundamental concepts and misunderstandings should be going on continually. This could be accomplished

by diagnostic tests given periodically and by the daily observations of the teacher who is constantly aware of the concepts and understandings necessary for progress in mathematics and who is alert in detecting such deficiencies. Whenever a weakness is discovered, remedial measures should be carried out. Such a plan would not be burdensome if all teachers in a system attempted to follow it.

CHAPTER V

SUMMARY AND CONCLUSIONS

The tests analyzed have been recognized as some of the best published, but they are not truly diagnostic. They were the best that could be obtained and they were administered over a period of three years which made it possible for the writer to trace some pupil difficulties that persisted. To carry out this kind of a study most exactly, the writer would have had to devise diagnostic tests for fundamental concepts and understandings, and to give them to a group of pupils over a period of three years.

The percentiles on standard tests formed a continuous series while teachers' marks formed a discrete series. Consequently, the comparison of ratings could not be as precise as the writer would like to have had it.

The questionnaire might have suggested dislike for mathematics through the repetition of the word, dislike, but granting that, it showed very clearly that a pupil who at some time develops a dislike for mathematics does not necessarily hold to that attitude thereafter in regard to succeeding mathematics courses.

The tables can be relied upon to show some general trends

CHAPTER V

SUMMARY AND CONCLUSIONS

in mathematical achievement for pupils of various degrees of intelligence in High School X. There is a group of pupils of high intelligence who are not achieving in mathematics according to their ability to achieve. The majority of the pupils of average intelligence achieve in mathematics according to the measured level of their intelligence. Many pupils with low intelligence quotients who pursue mathematics achieve more than their intelligence alone would warrant.

Dislike for mathematics is sporadic. A change in teacher, topic, or environment can change the attitude of a pupil toward mathematics.

Weaknesses in fundamental concepts and understandings in the cases sutdied persisted through the pupils' pursuit of mathematics causing an unreadiness for and a lack of foundation for the building of new mathematical ideas. Thus satisfactory progress in mathematics was impeded.

There is need for further research in the formation of truly diagnostic tests for checking fundamental concepts and understandings of mathematics and for checking high-school mathematics. There is need for research through experimental studies to compare the merit of teaching arithmetic through meanings with the merit of teaching arithmetic skills through drill.

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APPENDIX

IOWA EVERY-PUPIL TESTS OF BASIC SKILLS

ADVANCED BATTERY — GRADES 5-9

FORM O

By H. F. Spitzer

IN COLLABORATION WITH

Ernest Horn, Maude McBroom, H. A. Greene

AND E. F. Lindquist *(General Editor)*

all of the College of Education, State University of Iowa
with the Special Assistance of Julia Peterson and the Faculty of the
University Experimental Schools

TEST D: BASIC ARITHMETIC SKILLS

Directions: The other side of this page is an *answer sheet* on which you will mark your answers to all of the questions in this test. To use this answer sheet, you will have to tear it off. Do this now, tearing very carefully along the perforation at the left-hand side of this page.

* * * * *

(Tear off carefully along this perforation)

Each question in Part I of the test is followed by four possible answers, only one of which is correct or definitely better than any of the others. To answer a question, first decide which is the best answer, then look at the section labeled "Part I" in the upper left-hand corner of the answer sheet and find the *row* of boxes numbered the same as the *question*. Then place an **X** in one of these four boxes, as follows:

If you think the *first* answer is best, mark the *first* box in the row.

If you think the *second* answer is best, mark the *second* box in the row.

If you think the *third* answer is best, mark the *third* box in the row.

If you think the *fourth* answer is best, mark the *fourth* box in the row.

Mark only *one* box in each row. If you change your mind about an answer, erase your first mark very thoroughly.

Parts II and III of the test consist of examples and problems which you are to work. You will receive instructions for these parts later.

Answer the questions in the order in which they are given, but do not linger too long over difficult questions or problems. Skip them, and return to them later if time permits. If you do skip any questions, be sure to skip the corresponding boxes on the answer sheet also.

Do not begin work until you are told to do so.

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The Riverside Press Cambridge

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PAGES 1-2. PART I

ANSWER SHEET: ARITHMETIC TEST

1	☐☐☐☐	21	☐☐☐☐						
2	☐☐☐☐	22	☐☐☐☐						
3	☐☐☐☐	23	☐☐☐☐						
4	☐☐☐☐	24	☐☐☐☐						
5	☐☐☐☐	25	☐☐☐☐						
6	☐☐☐☐	26	☐☐☐☐						
7	☐☐☐☐	27	☐☐☐☐						
8	☐☐☐☐	28	☐☐☐☐						
9	☐☐☐☐	29	☐☐☐☐						
10	☐☐☐☐	30	☐☐☐☐						
11	☐☐☐☐	31	☐☐☐☐						
12	☐☐☐☐	32	☐☐☐☐						
13	☐☐☐☐	33	☐☐☐☐						
14	☐☐☐☐	34	☐☐☐☐						
15	☐☐☐☐	35	☐☐☐☐						
16	☐☐☐☐	36	☐☐☐☐						
17	☐☐☐☐	37	☐☐☐☐						
18	☐☐☐☐	38	☐☐☐☐						
19	☐☐☐☐	39	☐☐☐☐						
20	☐☐☐☐	40	☐☐☐☐						

NAME _____ SEX _____
(LAST NAME) (FIRST NAME) (BOY OR GIRL)

GRADE _____ AGE ON LAST BIRTHDAY _____ (YEARS) NUMBER OF FULL MONTHS SINCE LAST BIRTHDAY _____

SCHOOL _____ CITY _____

TEACHER _____ DATE _____

SCORES

Part	Poss. Score
I	(40) ____
II	(33) ____
III	(31) ____
Total	(104) ____

PAGES 3-4. PART II

Section A

Samples:

0	☐3	☒4	☐5	☐N
00	☐2	☐4	☐5	☒N

41	☐1916	☐2017	☐1918	☐N
42	☐1086	☐186	☐86	☐N
43	☐4340	☐42400	☐43500	☐N
44	☐$20\frac{3}{5}$	☐$2\frac{8}{13}$	☐21	☐N
45	☐130	☐129	☐132	☐N
46	☐1751	☐1641	☐1741	☐N
47	☐2437140	☐262740	☐2427140	☐N
48	☐$40\frac{8}{5}$	☐38	☐308	☐N
49	☐$\frac{7}{12}$	☐$1\frac{1}{6}$	☐$1\frac{1}{3}$	☐N
50	☐$36\frac{2}{3}$	☐$36\frac{1}{3}$	☐$37\frac{2}{3}$	☐N
51	☐$\frac{3}{5}$	☐$\frac{2}{5}$	☐$\frac{1}{3}$	☐N
52	☐$6\frac{3}{4}$	☐12	☐27	☐N
53	☐$12\frac{1}{2}$	☐$13\frac{1}{3}$	☐$12\frac{2}{3}$	☐N
54	☐$\frac{2}{3}$	☐$1\frac{2}{3}$	☐$1\frac{1}{3}$	☐N
55	☐28	☐30	☐60	☐N
56	☐$\frac{2}{7}$	☐7	☐$3\frac{1}{2}$	☐N
57	☐$8\frac{5}{24}$	☐$8\frac{11}{24}$	☐$8\frac{3}{8}$	☐N
58	☐$7\frac{17}{24}$	☐$8\frac{2}{3}$	☐$8\frac{17}{24}$	☐N
59	☐$\frac{1}{2}$	☐$\frac{9}{16}$	☐$\frac{3}{4}$	☐N
60	☐10	☐$15\frac{1}{2}$	☐$5\frac{4}{5}$	☐N
61	☐$8\frac{3}{4}$	☐$8\frac{1}{4}$	☐$8\frac{3}{8}$	☐N
62	☐$2\frac{1}{4}$	☐$1\frac{1}{4}$	☐$2\frac{1}{2}$	☐N
63	☐$10\frac{1}{2}$	☐$12\frac{3}{4}$	☐13	☐N

Section B

64	☐792	☐782	☐.792	☐N
65	☐1.22	☐122	☐12200	☐N
66	☐80	☐20	☐8000	☐N
67	☐80	☐800	☐25	☐N
68	☐$500	☐$50	☐$100	☐N
69	☐$\frac{3}{4}$%	☐25%	☐75%	☐N
70	☐$\frac{4.95}{1000}$	☐49.5%	☐4.95%	☐N
71	☐$\frac{76}{100}$	☐.076	☐.76	☐N
72	☐.9	☐$\frac{9}{10}$	☐$\frac{9}{100}$	☐N
73	☐$44.63	☐$44.60	☐$4463.00	☐N

PAGES 5-6. PART III

74	☐41	☐52	☐111	☐N
75	☐108	☐110	☐112	☐N
76	☐6	☐7	☐8	☐N
77	☐2	☐4	☐5	☐N
78	☐74	☐84	☐147	☐N
79	☐47	☐49	☐52	☐N
80	☐$1.36	☐$1.62	☐$1.72	☐N
81	☐$.40	☐$4.00	☐$6.00	☐N
82	☐40%	☐50%	☐60%	☐N
83	☐$7\frac{1}{2}$¢	☐11¢	☐$12\frac{1}{2}$¢	☐N
84	☐3	☐60	☐120	☐N
85	☐12	☐20	☐32	☐N
86	☐2,500 tons	☐3,000 tons	☐6,000 tons	☐50,000 tons

87 ☐3 times ☐4 times ☐5 times ☐6 times

88 ☐105 per cent ☐one-half ☐two-thirds ☐three-fourths

89 ☐one-third ☐one-fifth ☐one-tenth ☐three-fourths

90 ☐6 hours ☐$7\frac{1}{2}$ hours ☐$8\frac{1}{2}$ hours ☐9 hours

91 ☐40 miles ☐47 miles ☐50 miles ☐N

92 ☐3 miles ☐6 miles ☐7 miles ☐10 miles

93 ☐15% ☐40% ☐55% ☐75%

94 ☐$132,000 ☐$1,300,000 ☐$1,320,000 ☐N

95 ☐7 miles ☐20 miles ☐91 miles ☐N

96 ☐96 ☐97 ☐99 ☐N

97 ☐51 sq. mi. ☐120 sq. mi. ☐240 sq. mi. ☐480 sq. mi.

98 ☐108 sq. ft. ☐680 sq. ft. ☐720 sq. ft. ☐N

99 ☐9 feet ☐108 feet ☐20 feet ☐N

100 ☐$129.20 ☐$387.60 ☐$1162.80 ☐N

101 ☐$\frac{1}{18}$ ☐$\frac{1}{16}$ ☐$\frac{1}{8}$ ☐$\frac{1}{5}$

102 ☐None ☐$1.50 ☐$.50 ☐N

103 ☐$185 ☐$260 ☐$285 ☐N

104 ☐$9 ☐$15 ☐$18 ☐$22

Pupil's Name.....

PART I

VOCABULARY AND FUNDAMENTAL KNOWLEDGE

Directions: After each question in this part of the test there are four possible answers, only one of which is correct or definitely better than the others. Decide which is the best answer and then place an **X** in the proper box on the answer sheet.

1. 75 minutes is how many hours?
1) $\frac{3}{4}$ 2) $1\frac{1}{4}$ 3) $1\frac{1}{2}$ 4) $2\frac{1}{2}$

2. How should two hundred twenty-two and thirteen thousandths be written?
1) 20022.13 3) 222.13000
2) 222 13000 4) 222.013

3. About how high is an average dining table?
1) $2\frac{1}{2}$ feet. 3) 4 feet.
2) $3\frac{1}{2}$ feet. 4) 5 feet.

4. Which of these represents the largest value?
1) .6 2) .400 3) .3841 4) .0893

5. Which of these is used in measuring an angle?
1) Meters. 3) Degrees.
2) Cubic feet. 4) Centimeters.

6. Two and a half hours after midnight would be what time?
1) 12 A.M. 3) 9:30 P.M.
2) 2:30 P.M. 4) 2:30 A.M.

7. How should 5″ be read?
1) Five feet. 3) Five inches.
2) Five degrees. 4) Five hours.

8. Which of these fractions is the largest?
1) $\frac{5}{12}$ 2) $\frac{11}{18}$ 3) $\frac{12}{25}$ 4) $\frac{1}{3}$

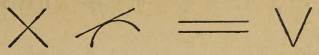
9. How many square feet are there in a square yard?
1) 3 2) 4 3) 6 4) 9

10. How many faces or sides does a cube have?
1) 4 2) 6 3) 8 4) 12

11. How would you read 100.001?
1) One hundred and one.
2) One hundred and one-tenth.
3) One hundred and one-hundredth.
4) One hundred and one-thousandth.

12. A ton of coal is about equal in weight to how many men?
1) 4 2) 9 3) 13 4) 20

13. In which of these figures is there a horizontal line?

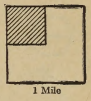


1) 2) 3) 4)

14. In which of the figures above do the lines form a right angle?
1) 1 2) 2 3) 3 4) 4

15. How many digits are used in writing the number four hundred twenty thousand seven?
1) 3 2) 5 3) 6 4) 9

16. About how many acres are in the shaded area in this diagram?



1) About $\frac{1}{4}$.
2) About 16.
3) About 160.
4) About 320.

17. How many $\frac{1}{6}$'s are in $\frac{2}{3}$?
1) Less than one. 3) 3
2) 2 4) 4

18. A tree 24 feet high is about how many times as high as a tall man?
1) 2 2) 3 3) 4 4) 6

19. Which of the following represents the largest quantity?
1) M 2) C 3) XL 4) XII

20. Why do we write the zero in 3.05?
1) Because arithmetic books say we should.
2) Because it holds the tenths place and shows that the 5 means 5 one-hundredths.
3) Because it shows that there are no fractions in the number.
4) Because the tenths place is always a zero when there are hundredths in a number.

21. 1000 B.C. is about how many years ago?
1) 1000 2) 940 3) 1940 4) 2940

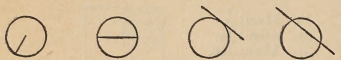
22. In looking at three groups of calves, one man said, "There are 6 in the first group, 6 in the second, and 8 in the third." A second man said, "There are 20 calves." If you only wanted to know how many calves there were, why was the second man's answer best?
1) Because it is easier to think of one group of 20 than of three groups of 6, 6, and 8.
2) Because 20 tells you how many calves there were.
3) Because 20 does not leave out any of the calves.
4) Because the first man did not tell how many calves there were.

(Go on to the next page.)

23. Which is equal to 4%?
1) $\frac{4}{10}$ 2) $\frac{1}{4}$ 3) $\frac{4}{100}$ 4) .40

24. In telling how long a certain bridge is, four children gave the following answers. Each answer is correct, but one is better than any other. Which is best?
1) About three times the distance across the school lawn.
2) About 40 times the length of this room.
3) About 12 times as far as the distance around the school room.
4) A person can run across it in about 2 minutes.

25. Which of these shows a diameter?



1) 2) 3) 4)

26. The length of the air field runway is 1800 feet. How many miles is this?
1) Less than half a mile. 3) 1 mile.
2) $\frac{1}{2}$ mile. 4) 2 miles.

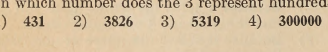
27. Which line is the circumference of the circle?

1) A
2) B
3) C
4) D

28. In which number does the 3 represent hundreds?
1) 431 2) 3826 3) 5319 4) 300000

29. In the number 555, how does the first 5 compare in value with the last 5?
1) It is the same.
2) It is twice as great.
3) It is 10 times as great.
4) It is 100 times as great.

30. Which of these figures shows what $\frac{1}{4} \times \frac{1}{2}$ equals?



1) 2) 3) 4)

31. In the word "eighty-one," what does the "ty" mean?
1) It is used to make the word sound rhythmical.
2) It means tens.
3) It means to add 80 and 1 together.
4) It means less than nine and more than eight.

32. What is the perimeter of a rectangle?
1) The distance around it.
2) Its area.
3) The distance from one corner to the opposite corner.
4) One-half the base times the altitude.

33. The population of city A is 161,832; that of city B is 43,126. What is the best way of expressing the relationship between the two populations?
1) A has 118,706 more people than B.
2) A is many times larger than B.
3) A is about four times as large as B.
4) A is about six times as large as B.

34. In what units would the volume of a box be given?
1) In centimeters.
2) In square inches.
3) In degrees.
4) In cubic inches.

35. In which of these situations would π be used? ($\pi = 3.14$)
1) In finding the thickness of a tree.
2) In finding the area of a triangle.
3) In finding the perimeter of a hexagon.
4) In finding the volume of a cube.

36. Which of these is a measure of area?
1) An acre. 3) A peck.
2) A rod. 4) A cubic foot.

37. In the last election, candidate A beat candidate B "two to one." If A received about 15,000 votes, approximately how many votes did B receive?
1) 7,500 3) 30,000
2) 10,000 4) 45,000

38. About how long would it take an eighth grade boy walking at a fast rate to walk a mile?
1) 5 minutes. 3) $\frac{1}{2}$ hour.
2) 15 minutes. 4) 1 hour.

39. If a farmer asks for the capacity of a grain bin, what units of measurement should a salesman use in answering?
1) Gallons. 3) Bushels.
2) Cubic feet. 4) Tons.

40. About how many 850-pound steers can be hauled in a truck with a load limit of 5 tons?
1) 6 2) 11 3) 16 4) 20

(Do not turn to the next page until you are told to do so.)

PART II

SECTION A: WHOLE NUMBERS AND FRACTIONS

Directions: In this section are a number of examples which you are to solve. On the answer sheet, after the number of each example, you will find three possible answers and an N. The N means that the correct answer for the example is not given on the answer sheet.

You are to do your work right on this page. First solve the example. Then turn to the answer sheet and find the number of that example. Compare your answer with those given on the answer sheet. If one of the three answers given is exactly like your own, place an **X** in the box in front of it. If none of the three answers given is like yours, place an **X** in the box in front of the N. Do not rework a problem simply because your answer is not like any of those given on the answer sheet. Instead, mark the box in front of the N and go on to the next example.

Reduce all fractions to *simplest* form.

The two sample items have been marked correctly on the answer sheet.

Sample 0 Add

$$\begin{array}{r} 2 \\ 2 \\ \hline 4 \end{array}$$

Sample 00 Subtract

$$\begin{array}{r} 9 \\ 3 \\ \hline 6 \end{array}$$

41 Add

$$\begin{array}{r} 706 \\ 394 \\ 817 \\ \hline \end{array}$$

42 Subtract

$$\begin{array}{r} 3005 \\ 2919 \\ \hline \end{array}$$

43 Multiply

$$\begin{array}{r} 217 \\ 200 \\ \hline \end{array}$$

44 Divide

$$52\overline{)1072}$$

45 Add

$$\begin{array}{r} 4 \\ 16 \\ 20 \\ 7 \\ 15 \\ 9 \\ 30 \\ 31 \\ \hline \end{array}$$

46 Subtract

$$\begin{array}{r} 4736 \\ 2089 \\ \hline \end{array}$$

47 Multiply

$$\begin{array}{r} 3020 \\ 807 \\ \hline \end{array}$$

48 Divide

$$85\overline{)3230}$$

49 Add

$$\frac{5}{6}$$

$$\frac{2}{6}$$

50 Subtract

$$\begin{array}{r} 42 \\ 5\frac{1}{3} \\ \hline \end{array}$$

51 Multiply

$$\frac{2}{3} \times \frac{1}{2} =$$

52 Divide

$$9 \div \frac{3}{4} =$$

53 Add

$$5\frac{5}{6}$$

$$7\frac{2}{3}$$

(Go on to the next page.)

54 Subtract

$8\frac{1}{3}$

$7\frac{2}{3}$

---

55 Multiply

$12 \times 2\frac{1}{3} =$

56 Divide

$\frac{7}{12} \div \frac{1}{6} =$

57 Add

$3\frac{2}{3}$

$\frac{1}{8}$

$4\frac{2}{3}$

---

58 Subtract

$13\frac{7}{8}$

$5\frac{1}{6}$

---

59 Multiply

$\frac{2}{3} \times 1\frac{1}{2} \times \frac{3}{4} =$

60 Divide

$6\frac{1}{5} \div \frac{2}{5} =$

61 Subtract

$10\frac{5}{8}$

$1\frac{7}{8}$

---

62 Subtract

31

$29\frac{3}{4}$

---

63 Divide

$8\frac{1}{2} \div \frac{2}{3} =$

(*Go on to Section B*)

SECTION B: PERCENTAGE AND DECIMALS

Directions: Do your work on this page if there is room; otherwise, use scratch paper. Mark the proper box on the answer sheet as you did in Section A.

64 Multiply .66 by .12.

65 Divide 244 by .02.

66 What is 20% of 400?

67 16 is 50% of what number?

68 What is 200% of $250?

69 Change $\frac{3}{4}$ to per cent form.

70 Change .495 to per cent form.

71 Change 76% to decimal form.

72 Change 90% to a common fraction and reduce it to its lowest terms.

73 What is 5% of $892.60?

(*Do not turn to the next page until you are told to do so.*)

PART III. PROBLEMS

Directions: Read each problem carefully. Do your work on scratch paper. Compare your answer with those given on the answer sheet and mark the proper box, as you did in Part II.

In some problems, you are asked to give only an *approximate* answer. For these particular problems, no N is given on the answer sheet, but you are to mark the box in front of the answer that is *most nearly* like your own.

At the beginning of the year, there were 13 girls and 18 boys in the third grade, 15 girls and 12 boys in the fourth grade, 11 girls and 16 boys in the fifth grade, and 13 girls and 13 boys in the sixth grade of the Jackson School.

74 How many girls were in the four grades?

75 How many more boys than girls were there in all four grades?

76 At the end of the year, there were 34 children in the fifth grade. How many more children were in the fifth grade at the end of the year than at the beginning?

77 The absences in the fifth grade during one week were as follows: Monday 3, Tuesday 0, Wednesday 5, Thursday 2, Friday 5. What was the average number of absences for each day?

On an automobile trip with his father, Tom kept a record of the speedometer readings as they drove along. At home it read 9209; at Salem the reading was 9217; at Vale City, 9291 miles; and at Greenville, 9356 miles.

78 How far was it from Salem to Vale City?

79 If it took 3 hours to make the trip from home to Greenville, how many miles per hour did they travel?

80 Before he started, Tom's father bought 8 gallons of gasoline at 17¢ per gallon and a quart of oil at 36¢ per quart. What was his bill?

The Girls' Club sold Christmas cards at $1.00 per box. The cards cost them 60¢ per box.

81 How much profit did they make on each box of cards they sold?

82 Their profit was what per cent of the selling price?

---

83 How much would it cost to send a letter weighing $2\frac{1}{2}$ ounces to Australia if postal rates are 5¢ for the first ounce and 3¢ for each additional ounce or fraction of an ounce?

84 How many tons of coal can be stored in a bin 4 feet wide, 10 feet long, and 3 feet deep? (Coal weighs about 50 pounds per cu. ft.)

85 The seventh grade planned to take a trip to an Indian reservation. The teacher said, "Mr. Brown is taking 5 of the children in his car, and I can take 3. That means we have rides for one-fourth of the class." How many children were in the seventh grade?

86 Ship A is rated as of 12,480 tons. If ship B is about one-fourth as large, what is its tonnage? (Note that in this problem no *exact* relationship is stated. Therefore, your answer will be only an approximation.)

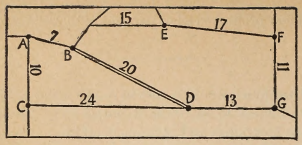
87 A certain airplane that has a top speed of 435 miles per hour is approximately how much faster than an automobile which has a top speed of 90 miles per hour? (Only an *approximate* answer is required.)

88 If a man plants 105 of his 160 acres in corn, about what part of his farm does he plant in corn? (Only an *approximate* answer is required.)

89 A dress in a store window has these two prices marked on it: "Was $12.98 — Now $10.25." About how much of a price reduction has been made? (Only an *approximate* answer is required.)

90 John is waiting for a train that is scheduled to arrive at 9:35 A.M. but has been marked 8 hours late. John looks at his watch and sees that it is 9:00 A.M. About how much longer must he wait for the train? (Only an *approximate* answer is required.)

(*Go on to the next page.*)



This is a section of a road map. The numbers between points indicate the number of miles between those points. The solid line indicates paved road. The double line indicates gravel road.

91 What is the shortest road distance from A to G?

92 In going from D to A, how many miles farther is it to go the all paved road than to go over part that is gravel?

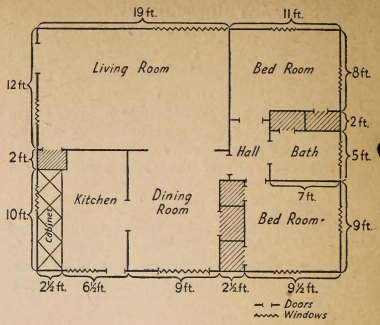
93 About what per cent of the most direct road from B to G is paved? (Only an *approximate* answer is required.)

94 If the cost of building a paved road is $55,000 per mile, what was the total cost of the road from C to D?

95 If the cost of building a gravel road is only $6,000 per mile, how many miles of gravel road can be built for the same amount of money that one mile of paved road costs ($55,000)?

96 On an auto trip, Mr. Brown goes from C to F by way of G. He returns by way of A. How many miles did he drive on the trip?

97 What is the approximate area in square miles of the region enclosed by the road from A to C to D and then back to A by way of B?



This is a simplified floor plan of a house. (You may consider the dimensions given as the inside dimensions of the room, and you need pay no attention to the thickness of walls and partitions.) The shaded areas represent space used for closets. Problems 98 to 104 are based on this diagram.

98 What is the total area of this floor plan?

99 The kitchen floor and cabinet top are to be covered with linoleum which comes only in 6-foot widths. How many feet of this 6-foot width material should be purchased?

100 The floor carpeting for the living room of this house costs $5.10 a square yard. What will be the cost of this carpeting?

101 About what fraction of the total area is used in closets? (Only an *approximate* answer required.)

102 The builder of this house thought that the estimate on the cost of doors was too high. The contractor pointed out that outside doors were $15 each, standard interior doors were $4.50 each, and closet doors were $3.00. The estimate for doors was $70.00. How much *too high* was this estimate?

103 If the large living room and dining room windows together cost $85.00 and the other windows cost $25.00 per unit, what was the cost of windows in this house?

104 The loan on this house is $5,000, on which the owner pays $30.00 per month. If the rate of interest is 5%, what is the approximate amount of the principal that is paid the first month? (Only an *approximate* answer is required.)

(Turn your booklet over and wait until the papers are collected.)

1942 Edition

AMERICAN COUNCIL ON EDUCATION

Psychological Examination

For High School Students

Prepared by L. L. Thurstone and Thelma Gwinn Thurstone

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General Instructions

This examination is different from the ordinary school examinations to which you have been accustomed. The plan for each of these tests is as follows. First, you are given detailed *instructions* about the test, so that you know just what you are expected to do. Then you have some *practice problems*. Then you go to the *test proper*. This is the procedure for each of the four tests in this examination.

The four tests in this examination represent a variety of tasks. Two of them involve thinking of a quantitative sort, while the other two require more linguistic ability. If you find one test hard, do not be discouraged. You may find the next test easier. But you should do your best on all the tests.

People differ markedly in the speed with which they can do these different tests. The tests are long enough to keep everyone busy for the whole time, and you are not expected to complete the tests in the time allowed. By noting how many questions you can answer in a certain length of time, we can determine your speed on each kind of test. You must begin to work on a test promptly when the examiner calls the starting time and stop immediately when he says: "Stop." Do not begin a test until the examiner gives the starting signal for that particular test. Do not turn back to a test after the time for it has expired. You are to work on each test during, and only during, the specified time as announced by the examiner in charge.

You are to record your answers on a separate answer sheet rather than on the pages of the test booklet. Instead of writing down your answers in the usual manner, you will record each answer by blackening the space between a pair of lines. *Do not make any marks or record any answers on the pages of this test booklet.*

Your answer sheet will be scored accurately if you observe carefully the following directions:

1. On the answer sheet, find the *section* which corresponds to the practice problems or test proper on which you are working.
2. Then find the *row of answer spaces* which is numbered the same as the question you are answering.
3. Then find the *pair of dotted lines* which corresponds to the answer you choose and blacken the space. MISPLACED ANSWERS ARE COUNTED AS WRONG ANSWERS.
4. Indicate each answer with SOLID BLACK PENCIL MARKS drawn vertically between the two dotted lines. Solid black marks are made by going over each mark two or three times and by pressing firmly on the pencil.
5. Make your marks as long as the dotted lines.
6. If you change your answer, erase your first mark completely.
7. Make no unnecessary marks in or around the dotted lines.
8. Keep your answer sheet on a hard surface while marking your answers.
9. Make no folds or creases in the answer sheets.
10. *No scratch paper* is allowed in any of these tests. The answer sheet contains a special section which may be used for scribbling.
11. Fold the pages of your test booklet back so that *only one page is visible*. Place the test booklet to the left. Keep the answer sheet under the test booklet so that the answer spaces being marked are as close as possible to the questions being answered.

(Omit the next paragraph unless the tests are to be machine-scored.)

The examination will be scored by an electric test-scoring machine, which makes use of the fact that a solid black pencil mark will carry a current of electricity in the same way that a copper wire does. LIGHT PENCIL MARKS MADE WITH A HARD PENCIL WILL NOT CARRY A CURRENT OF ELECTRICITY! The machine will not give you a correct score unless you indicate your answers with solid black pencil marks made with the *special* pencil which is provided. Do not use any pencil other than the special one provided. The machine cannot distinguish between intended answers and stray pencil marks. If you are careless in erasing, or if you leave unnecessary marks on or near the pairs of lines, such marks may be counted by the machine as wrong answers so that your score will be lower than it should be.

Wait until the examiner gives the starting signal for the first set of practice problems.

Same-Opposite

PRACTICE PROBLEMS

The word at the left in the following line is "many."

1. many	(1) ill	(2) few	(3) down	(4) sour

One of the four words at the right means either the *same* as or the *opposite* of "many." The word "few," which is numbered 2, is the opposite of "many." In the section of the answer sheet labeled "SAME-OPPOSITE, Practice Problems, Page 3," space number 2 in the first row has been blackened.

The word at the left in the second example is "ancient." Select one of the four words at the right that means the *same* as or the *opposite* of "ancient." In the second row on the answer sheet, blacken the space which corresponds to the answer you have selected.

2. ancient	(1) dry	(2) long	(3) happy	(4) old

You should have blackened the space numbered 4, because 4 corresponds to "old," which means the same as "ancient."

In each of the following lines select the word that means the *same* as or the *opposite* of the word at the left. On the answer sheet, blacken the space which corresponds to the answer you have selected.

3. deep	(1) blue	(2) shallow	(3) tense	(4) watery
4. awkward	(1) clumsy	(2) loyal	(3) passive	(4) young
5. hot	(1) dry	(2) cooked	(3) red	(4) cold

When the starting signal is given (not yet), turn the page and work more problems of the same kind. Work rapidly because your rating will be the total number of correct answers. You may not be able to finish in the time allowed.

Stop here. Wait for the signal.

In each row, select the word at the right which means the *same* as or the *opposite* of the first word in the row. Blacken the space which corresponds to the word you have selected.

SAME-OPPOSITE

1. agreeable	(1) new	(2) stalwart	(3) dark	(4) pleasant
2. legendary	(1) unselfish	(2) true	(3) pleasant	(4) noisy
3. elderly	(1) drab	(2) brotherly	(3) absurd	(4) youthful
4. seasonable	(1) dutiful	(2) dauntless	(3) timely	(4) gracious
5. tremendous	(1) brilliant	(2) crude	(3) slight	(4) past
6. nonchalant	(1) actual	(2) indifferent	(3) varied	(4) unruly
7. variable	(1) conquered	(2) shifting	(3) bitter	(4) sudden
8. respectful	(1) deferential	(2) physical	(3) remedial	(4) several
9. barbarous	(1) tidal	(2) haughty	(3) cultured	(4) abrupt
10. zestful	(1) stormy	(2) tough	(3) eager	(4) lengthy
11. important	(1) adequate	(2) dishonest	(3) trifling	(4) open
12. staunch	(1) cozy	(2) uneven	(3) unwavering	(4) stupid
13. brawny	(1) clever	(2) dim	(3) hazy	(4) puny
14. absolute	(1) ungainly	(2) eligible	(3) gaudy	(4) partial
15. raw	(1) silken	(2) slick	(3) cooked	(4) stale
16. aqueous	(1) literal	(2) watery	(3) manual	(4) informal
17. sublime	(1) liberal	(2) straight	(3) exalted	(4) brisk
18. immense	(1) prodigious	(2) bloody	(3) wistful	(4) vulgar
19. elastic	(1) trivial	(2) resilient	(3) valid	(4) humorous
20. ethical	(1) perishable	(2) moral	(3) eloquent	(4) garish
21. turbulent	(1) eastern	(2) selective	(3) interested	(4) peaceful
22. envious	(1) spicy	(2) brainless	(3) covetous	(4) vain
23. fastidious	(1) musical	(2) famed	(3) negligent	(4) early
24. sorrowful	(1) tolerant	(2) blunt	(3) unsteady	(4) blithe
25. transient	(1) glib	(2) sensitive	(3) surly	(4) fleeting
26. inimitable	(1) matchless	(2) optical	(3) outlined	(4) parental
27. austere	(1) narrow	(2) stern	(3) unhappy	(4) vile
28. evil	(1) heedless	(2) frantic	(3) volatile	(4) pernicious
29. skeptical	(1) vague	(2) believing	(3) constant	(4) unwise
30. incessant	(1) demandable	(2) intermittent	(3) foreign	(4) unhappy
31. suave	(1) prevalent	(2) neuter	(3) old	(4) brusque
32. romantic	(1) facial	(2) subdued	(3) judicial	(4) prosaic
33. energetic	(1) apathetic	(2) balmy	(3) criminal	(4) heroic
34. buoyant	(1) dejected	(2) bestial	(3) clear	(4) savage
35. cavernous	(1) matted	(2) smooth	(3) unsuitable	(4) hollow
36. effectual	(1) tired	(2) unarmed	(3) wide	(4) adequate
37. seemly	(1) poetic	(2) scribbled	(3) local	(4) fit
38. solicitous	(1) natural	(2) fearless	(3) thoughtful	(4) calm
39. intense	(1) likely	(2) supple	(3) vivid	(4) respectful
40. formidable	(1) mystic	(2) obscene	(3) menacing	(4) nodal
41. precocious	(1) nodding	(2) hairy	(3) endless	(4) backward
42. sinister	(1) auspicious	(2) settled	(3) diligent	(4) tacit
43. ignominious	(1) indecisive	(2) degrading	(3) forward	(4) inconstant
44. canonical	(1) flushed	(2) willful	(3) orthodox	(4) frozen
45. staid	(1) frivolous	(2) open	(3) tabular	(4) harsh
46. pungent	(1) sleepy	(2) bland	(3) doleful	(4) lavish
47. impeccable	(1) fearless	(2) grudging	(3) faultless	(4) secret
48. furtive	(1) drab	(2) rugged	(3) stealthy	(4) placid
49. cognizant	(1) next	(2) acquired	(3) unofficial	(4) unaware
50. ductile	(1) plenteous	(2) regional	(3) flexible	(4) silent

Stop here.

Completion

PRACTICE PROBLEMS

Look at the following definition. You are to think of the word that fits the definition.

1. A contest of speed.
B F M P R

The word is *race*. The letter *R* is the first letter in the word *race*. In the section of the answer sheet labeled "COMPLETION, Practice Problems, Page 5," the space indicated by *R* in the first row has been blackened.

Blacken the space corresponding to the first letter of the word which fits the following definition.

2. A place or building for athletic exercises.
C D G H T

The word is *gymnasium*. You should have marked G because it is the first letter in the word *gymnasium*.

Do the following examples in the same way:

3. The thin cutting part of an instrument, as of a knife or sword.
A B D H W

4. The wife of a king.
F N P Q V

5. A small or portable bed, as of canvas stretched on a frame.
C H N P T

When the starting signal is given (not yet), turn the page and work more problems of the same kind. Work rapidly because your rating will be the total number of correct answers. You may not be able to finish in the time allowed.

Stop here. Wait for the signal.

Think of the word that fits the definition. Then mark the first letter of that word on the answer sheet.

COMPLETION

1. A fertile or green spot in a waste or desert.
L N O T U

2. The missile weapon used with a bow.
A G N R U

3. The part of a house appropriated to the care of children.
D F H L N

4. A feast, often ceremonious and followed by speeches.
A B E H J

5. The early part of the day.
C H J M P

6. Time that is to come.
B D F G I

7. The sixtieth part of an hour.
L M N O P

8. Explanation of the meaning of a word.
A B D F J

9. A fastening, as for a door, operated by a key.
C G I J L

10. A period of one hundred years.
B C D F G

11. A fluid preparation, as of the essence of flowers, used for scenting.
M P R U V

12. A seat for a rider on a horse's back.
C D J R S

13. A missile, usually of lead, to be shot from a small firearm.
A B D E H

14. A small elementary book for teaching children to read.
D F K L P

15. An enclosing border of a picture.
A C D F I

16. One who violates his allegiance and betrays his country.
D G P T U

17. A place devoted to experimental study in any science.
A D E F L

18. The price of transportation for a person.
B F H K O

19. A stopper for a bottle to prevent the flow of liquid.
C D G H M

20. A bride's personal outfit, as of clothes, jewelry, etc.
E G K M T

21. Memoirs of one's life written by oneself.
A F K S U

22. A thickly populated street marked by wretched living conditions.
F J S U V

23. A child in the first period of life.
G H I L M

24. A human head represented sidewise.
H N P R W

25. A roundabout way temporarily replacing part of a route.
C D G O V

26. A person who retires from society and lives in solitude.
B D F H K

27. One who performs daring gymnastic feats.
A E I O U

28. The official residence of a sovereign.
D E J O P

29. A first public appearance, as of an actor.
A C D H M

30. One who travels to some holy place as a devotee.
A F H N P

Think of the word that fits the definition. Then mark the first letter of that word on the answer sheet.

COMPLETION

31. One who enters into, or offers himself for, any service of his own free will.
D J K U V

32. A three-wheeled vehicle propelled by pedals.
D F N T W

33. The long projecting nose of a beast, as of swine.
L R S U V

34. An instrument used for looking out over the water from a submerged submarine.
A G N P W

35. A person, animal, or plant that has reached maturity.
A C I N T

36. An imaginary circle dividing the earth's surface into the northern and southern hemispheres.
A B D E F

37. A distinguishing mark, device, or symbol used on merchandise.
A G J T V

38. A lure to catch fish or other animals.
B J K O V

39. The working room of a painter or sculptor.
C J K P S

40. A glass in one compartment of a window frame.
C E H P T

41. A visible mass of fog or haze suspended at a height in the air.
B C G L O

42. The malicious burning of property.
A E K O U

43. Prolonged inability to obtain due sleep.
G H I J K

44. Recovery, especially gradual recovery, of health after sickness.
A B C D E

45. A unit of weight for precious stones, especially diamonds and pearls.
A C H N T

46. A very large funnel used as a speaking trumpet.
E J M N R

47. The natural abode of an animal or plant.
B H I J U

48. One without means except such as come from charity.
A K P U Y

49. The spirit, usage, or manners of knighthood.
A C D I N

50. One who is inclined to put the least favorable construction upon actions and happenings.
H J P R S

51. A kind of very large nail.
C I S U W

52. A light spear for hurling.
C F H J K

53. The resistance to motion between two surfaces in contact.
B C D E F

54. A specified or regular course of study.
C M N O T

55. A short fictitious narrative from which a moral or spiritual truth is drawn.
G H N P U

56. A small shrill flute.
E H J M P

57. One who works in stone.
D F M R Y

58. A representation of the outlines of an object filled in with some uniform color.
G L N S W

59. A soft pillow or pad to rest on or against.
A C F L O

60. The mass of leafage of a plant as produced in nature.
A E F J R

Stop here.

Arithmetic

PRACTICE PROBLEMS

In this test you will be given some problems in arithmetic. After each problem there are five answers, but only one of them is the correct answer. You are to solve each problem and blacken the space on the answer sheet which corresponds to the answer you think is correct. The following problem is an example.

1. How many pencils can you buy for 50 cents at the rate of 2 for 5 cents?
(a) 10 (b) 20 (c) 25 (d) 100 (e) 125

Find on the answer sheet the space labeled "ARITHMETIC, Practice Problems, Page 9." The correct answer to the problem is 20, which is answer (b).
In the row numbered 1, space (b) has been blackened.

In the *second* row, blacken the space which corresponds to the answer to the second practice problem.

2. If James had 4 times as much money as George, he would have $16. How much money has George?
(a) $4 (b) $8 (c) $12 (d) $16 (e) $64

You should have blackened space (a), which corresponds to $4, the correct answer.

Blacken the spaces corresponding to the answers to the following problems:

3. In 5 days Harry has saved a dollar. What has his average daily saving been?
(a) 20¢ (b) 22½¢ (c) 25¢ (d) 30¢ (e) 40¢

4. John sold 4 magazines at 5 cents each. He kept ½ the money and with the other ½ he bought papers at 2 cents each. How many did he buy?
(a) 3 (b) 4 (c) 5 (d) 6 (e) 10

When the signal is given (not yet), turn the page and work more problems of the same kind. Work rapidly and accurately. Your rating will be the total number of correct answers. You may not be able to finish in the time allowed.

Stop here. Wait for the signal.

Find the correct answer to each problem below. Then blacken the corresponding space on the answer sheet.

ARITHMETIC

1. Twelve girls rented a cottage for 3 months at $40 per month. What was the total rent paid by each girl?
(a) $3.33 (b) $9.00 (c) $10.00 (d) $12.66 (e) $120.00

2. If 3½ tons of coal cost $21, what will 7½ tons cost?
(a) $9.80 (b) $42 (c) $45 (d) $75 (e) $98

3. A has $320, B has ¼ as much as A, and C has ½ as much as A and B together. How much have all together?
(a) $400 (b) $480 (c) $500 (d) $520 (e) $600

4. A tank which holds 260 gallons of oil is ¼ full. How many gallons of oil are needed to fill the tank?
(a) 65 (b) 195 (c) 200 (d) 205 (e) 220

5. A boy walked for 2¼ hours. He started at 10 minutes to 9 o'clock. When did he finish?
(a) 10:35 (b) 11:05 (c) 11:20 (d) 11:55 (e) 12:05

6. Ray raised 60 heads of cabbage averaging 5 pounds each. He sold them at $.03 a pound. He spent $1.25 for plants and $.58 for fertilizer. What was his profit?
(a) $6.17 (b) $6.67 (c) $7.17 (d) $7.27 (e) $7.87

7. If it takes 8 barrels of oil at $1.35 per barrel to sprinkle ½ mile of road, how much will oil cost for 5 miles?
(a) $27 (b) $54 (c) $81 (d) $90 (e) $108

8. In a fort there are 60 men and enough food to keep the 60 men for 20 days. If 20 new men come and 40 of the first go, how many days will the food last?
(a) 10 (b) 20 (c) 30 (d) 40 (e) 60

9. A grocer bought 160 boxes of berries. From the first 8 boxes examined he had to throw away 1 box. At this rate, how many boxes will he be able to sell?
(a) 20 (b) 21 (c) 120 (d) 139 (e) 140

10. A U-boat makes 8 miles an hour under water and 15 miles an hour on the surface. How many hours will it take to cross a 100-mile channel, if it has to go ⅖ of the way under water?
(a) 4 (b) 5 (c) 6 (d) 8 (e) 9

11. Two carpenters received $150 for the work they did on a house. One worked 20 days and the other 30 days. How much more did one get than the other?
(a) $30 (b) $35 (c) $40 (d) $45 (e) $50

12. Soldiers march 2 feet 6 inches per step and take 100 steps to the minute. How many feet do they march in $\frac{1}{12}$ of an hour?
(a) 1000 (b) 1250 (c) 1320 (d) 1500 (e) 1600

13. If ⅔ of a yard of silk costs $3, how many yards can be bought for $22.50?
(a) 4 (b) 5 (c) 5½ (d) 5⅔ (e) 7½

14. John has 4 times as many marbles as James, and together they have 75 marbles. If Henry buys ⅕ of John's marbles and 33⅓ per cent of James', how many will he gain?
(a) 10 (b) 12 (c) 15 (d) 17 (e) 23

15. The average person attends school 1,080 days. What part of a 12-year course does he complete, counting 180 days as a school year?
(a) ⅓ (b) ½ (c) ⅝ (d) ⅔ (e) ¾

16. A fruit dealer buys 10 dozen oranges for $2.40. If two dozen spoil, at what price per dozen must he sell the good ones to gain ⅛ of the whole cost?
(a) $.20 (b) $.25 (c) $.30 (d) $.33 (e) $.40

17. If a fowl loses ¼ in dressing, how many pounds of undressed fowl will be necessary to dress 9 pounds?
(a) 12 (b) 12½ (c) 13½ (d) 15 (e) 18

18. A can do a piece of work in 8 days, while B would take 20 days. After A has worked alone for 3 days, how many days will it take B to finish the work?
(a) 8 (b) 9 (c) 10 (d) 12 (e) 12½

19. Five lamp posts are placed along a street 35 yards apart. How many yards is the first lamp post from the last?
(a) 95 (b) 140 (c) 175 (d) 420 (e) 525

20. If ¾ of A's money equals ½ of B's money, and they have $40 together, how much has A?
(a) $8 (b) $10 (c) $12 (d) $16 (e) $24

Stop here.

Number Series

PRACTICE PROBLEMS

The numbers in each series proceed according to some rule. For each series you are to find the *next number*.

In the first series below, each number is 2 larger than the preceding number. The *next number* in the series would be 14. Of the five answers at the right, answer (e) is, therefore, correct. In the section of the answer sheet labeled "NUMBER SERIES, Practice Problems, Page 11," space (e) in the first row has been blackened.

	Series						*Next Number*				
1.	2	4	6	8	10	12	10 (a)	11 (b)	12 (c)	13 (d)	14 (e)

Find the rule in the series below, and blacken one of the answer spaces in the second row on the answer sheet.

2.	20	19	18	17	16	15	10 (a)	12 (b)	14 (c)	15 (d)	16 (e)

Each number in this series is 1 less than the preceding number. You should have blackened space (c), which corresponds to 14, the next number in the series.

Find the rule in the series below, and blacken the space on the answer sheet which corresponds to the next number.

3.	10	8	11	9	12	10	9 (a)	10 (b)	11 (c)	12 (d)	13 (e)

The series above goes by alternate steps of subtracting 2 and adding 3. You should have blackened space (e), which corresponds to 13, the next number.

In each series below, find the rule and blacken the space on the answer sheet which corresponds to the next number. There is a different rule for each series. Go right ahead. Do not wait for any signal.

4.	8	11	14	17	20	23	10 (a)	13 (b)	23 (c)	25 (d)	26 (e)
5.	27	27	23	23	19	19	15 (a)	16 (b)	17 (c)	18 (d)	19 (e)
6.	16	17	19	20	22	23	18 (a)	20 (b)	22 (c)	24 (d)	25 (e)

When the starting signal is given (not yet), turn the page and work more problems of the same kind. Work rapidly because your rating will be the total number of correct answers. You may not be able to finish in the time allowed.

Stop here. Wait for the signal.

Find the rule in each problem below and blacken the space which corresponds to the next number.

NUMBER SERIES

1.	14	18	22	26	30	34	38	39 (a)	40 (b)	41 (c)	42 (d)	43 (e)	**16.**	100	90	81	73	66	60	55	46 (a)	50 (b)	51 (c)	52 (d)	53 (e)
2.	7	10	9	12	11	14	13	12 (a)	14 (b)	16 (c)	18 (d)	20 (e)	**17.**	12	6	8	16	14	7	9	7 (a)	11 (b)	12 (c)	18 (d)	36 (e)
3.	17	20	23	26	29	32	35	36 (a)	37 (b)	38 (c)	39 (d)	40 (e)	**18.**	4	8	9	18	22	23	46	48 (a)	50 (b)	69 (c)	70 (d)	90 (e)
4.	8	12	11	15	14	18	17	16 (a)	18 (b)	20 (c)	21 (d)	24 (e)	**19.**	40	42	21	24	8	12	3	1 (a)	4 (b)	6 (c)	7 (d)	8 (e)
5.	20	18	21	17	22	16	23	9 (a)	12 (b)	15 (c)	21 (d)	24 (e)	**20.**	9	12	8	10	13	9	11	10 (a)	11 (b)	14 (c)	16 (d)	17 (e)
6.	72	36	40	20	24	12	16	4 (a)	8 (b)	12 (c)	16 (d)	20 (e)	**21.**	40	33	27	21	16	11	7	1 (a)	2 (b)	3 (c)	5 (d)	7 (e)
7.	8	6	4	12	10	8	16	6 (a)	10 (b)	12 (c)	14 (d)	16 (e)	**22.**	4	5	7	4	8	13	7	14 (a)	15 (b)	16 (c)	21 (d)	22 (e)
8.	2	4	12	14	42	44	132	133 (a)	134 (b)	260 (c)	268 (d)	396 (e)	**23.**	44	40	42	14	10	12	4	0 (a)	2 (b)	6 (c)	7 (d)	10 (e)
9.	82	73	64	55	46	37	28	14 (a)	18 (b)	19 (c)	20 (d)	27 (e)	**24.**	2	3	5	5	10	11	13	13 (a)	15 (b)	16 (c)	26 (d)	28 (e)
10.	0	1	3	6	10	15	21	23 (a)	25 (b)	28 (c)	29 (d)	30 (e)	**25.**	20	2	12	60	6	16	80	2 (a)	4 (b)	6 (c)	8 (d)	100 (e)
11.	4	5	6	7	5	6	7	3 (a)	6 (b)	7 (c)	8 (d)	9 (e)	**26.**	4	6	3	7	9	6	10	4 (a)	7 (b)	12 (c)	14 (d)	16 (e)
12.	18	20	17	21	16	22	15	8 (a)	17 (b)	21 (c)	23 (d)	30 (e)	**27.**	10	6	24	28	7	3	12	3 (a)	6 (b)	16 (c)	24 (d)	48 (e)
13.	8	9	12	13	15	16	19	17 (a)	20 (b)	21 (c)	23 (d)	24 (e)	**28.**	7	9	12	8	3	9	16	8 (a)	9 (b)	19 (c)	23 (d)	24 (e)
14.	20	16	8	24	20	10	30	15 (a)	23 (b)	26 (c)	28 (d)	90 (e)	**29.**	95	92	46	42	21	16	8	2 (a)	4 (b)	6 (c)	8 (d)	10 (e)
15.	7	4	12	15	5	2	6	0 (a)	1 (b)	5 (c)	6 (d)	9 (e)	**30.**	9	3	8	4	7	5	6	4 (a)	5 (b)	6 (c)	7 (d)	8 (e)

Stop here.

AMERICAN COUNCIL ON EDUCATION

COOPERATIVE ALGEBRA TEST

ELEMENTARY ALGEBRA THROUGH QUADRATICS
REVISED SERIES FORM R

by

JOHN A. LONG, University of Toronto; L. P. SICELOFF, Columbia University;
and LEONE E. CHESIRE, Cooperative Test Service

with the editorial assistance of

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Please print:

Name.....Date.....
Last First Middle

Grade or Class.....Age.....Date of Birth.....
Yrs. Mos.

School.....City.....Sex.....
M. or F.

Title of the algebra course you are now taking.....Instructor.....

In what grade did you begin the study of algebra?.....

Number of years you have studied algebra (one semester = ½ year; one quarter = ⅓ year):.....

General Directions: Do not turn this page until the examiner tells you to do so. This examination consists of three parts, and requires 40 minutes of working time. The directions for each part are printed at the beginning of the part. Read them carefully, and proceed at once to answer the questions. DO NOT SPEND TOO MUCH TIME ON ANY ONE ITEM. ANSWER THE EASIER QUESTIONS FIRST; then return to the harder ones if you have time. There is a time limit for each part. You are not expected to answer all the questions in any part in the time limit; but if you should, go on to the next part. If you have not finished Part I when the time is up, stop work on that part and proceed at once to Part II. If you finish the last part before the time is up, you may go back and work on any part. No questions may be asked after the examination has begun.

You may answer questions even when you are not perfectly sure that your answers are correct, but you should avoid **wild** guessing, since wrong answers will result in a subtraction from the number of your correct answers.

Part	I	II	III	Total
Minutes	15	10	15	40

Scaled Score	Percentile

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PART I

Directions: Each problem below is followed by five answers, only one of which is correct. By working each problem find the correct answer and put its **number** in the parentheses at the right.

1. $5a - 3a$ equals
   1–1 $-2a$
   1–2 $2a$
   1–3 $-8a$
   1–4 $8a$
   1–5 $-15a$ . . . . . .1( )

2. $\frac{r}{r}$ equals
   2–1 1
   2–2 r
   2–3 $\frac{1}{2}r$
   2–4 0
   2–5 r^2 . . . . . .2( )

3. What is the quotient when $-12b^{10}c^4$ is divided by $-6b^2c^4$?
   3–1 $2b^8$
   3–2 $-2b^5$
   3–3 $2b^5$
   3–4 $-2b^8$
   3–5 $2b^8c$ . . . . . .3( )

4. What is the sum of $3.5c$ and $21.34c$?
   4–1 $13.66c$
   4–2 $17.84c$
   4–3 $21.69c$
   4–4 $24.84c$
   4–5 $74.69c^2$ . . . . . .4( )

5. What is the result when $-13b^4$ is subtracted from $5b^4$?
   5–1 $-18b^8$
   5–2 $18b^4$
   5–3 $8b^4$
   5–4 $-8b^4$
   5–5 $-8b^8$ . . . . . .5( )

6. Express $b^2 - 16$ as the product of two factors.
   6–1 $(b-4)(b-4)$
   6–2 $(b-4)(b+4)$
   6–3 $(b+4)(b+4)$
   6–4 $(b+2)(b-8)$
   6–5 $(b-1)(b+16)$ . . . . . .6( )

7. If $\frac{5m+1}{4} = 9$, what does m equal?
   7–1 $6\frac{2}{5}$
   7–2 7
   7–3 $7\frac{2}{5}$
   7–4 8
   7–5 11 . . . . . .7( )

8. If $7n - 5p = 2$, what is the value of p when n equals 6?
   8–1 8
   8–2 -8
   8–3 $8\frac{4}{5}$
   8–4 4
   8–5 $4\frac{4}{7}$ . . . . . .8( )

9. What is the product of $3n^4$ and $4n^2$?
   9–1 $\frac{3}{4}n^2$
   9–2 $\frac{4}{3}n^2$
   9–3 $7n^6$
   9–4 $12n^6$
   9–5 $12n^8$ . . . . . .9( )

10. Simplify $17n + 5 - 3(4n - 3)$ by removing the parentheses and combining like terms.
    10–1 $29n + 14$
    10–2 $13n + 2$
    10–3 $5n + 14$
    10–4 $5n + 8$
    10–5 $5n - 4$ . . . . . .10( )

11. If $r = 2$, $s = -1$, and $t = -3$, what is the numerical value of $4r - 3s + 2t$?
    11–1 -1
    11–2 11
    11–3 17
    11–4 8
    11–5 5 . . . . . .11( )

12. If $5k - 1 - (3k + 4) = 9$, what does k equal?
    12–1 $1\frac{1}{2}$
    12–2 2
    12–3 3
    12–4 6
    12–5 7 . . . . . .12( )

Go on to the next page.

13. If $\frac{4m+5}{3} = \frac{7m+6}{5}$, what does **m** equal?

13–1 $1\frac{2}{41}$

13–2 $\frac{1}{15}$

13–3 -7

13–4 7

13–5 **43** . . . . . 13( )

14. The relation between **k** and **n** is indicated by the formula $k = \frac{3}{2}n + 50$. What is the value of **k** when **n** equals -30?

14–1 95

14–2 $-53\frac{1}{3}$

14–3 -45

14–4 30

14–5 **5** . . . . . 14( )

15. Combine $8\sqrt{7} - 3\sqrt{7} + \sqrt{7}$.

15–1 6

15–2 $6\sqrt{7}$

15–3 $12\sqrt{7}$

15–4 $5\sqrt{7}$

15–5 **5** . . . . . 15( )

16. If $4s + t = 40$ and $s - t = 5$, what does **s** equal?

16–1 15

16–2 $11\frac{2}{3}$

16–3 9

16–4 8

16–5 **7** . . . . . 16( )

17. If the graph of the equation $4x + 3y = 3$ passes through the point $(k, -7)$, what is the value of **k**?

17–1 $10\frac{1}{3}$

17–2 $-8\frac{1}{3}$

17–3 6

17–4 8

17–5 $-4\frac{1}{2}$ . . . . . 17( )

18. Change $\frac{3an - 9a^2}{n^2 - 9a^2}$ to lowest terms.

18–1 $\frac{3a}{n - 3a}$

18–2 $\frac{3a}{n + 3a}$

18–3 $\frac{1}{n}$

18–4 $\frac{3a}{n}$

18–5 $\frac{n - 3a}{n(n + 3a)}$ . . . . . 18( )

19. What is the positive square root of $64x^{16}$?

19–1 $8x^8$

19–2 $8x^{16}$

19–3 $8x^4$

19–4 $4x^4$

19–5 $16x^4$ . . . . . 19( )

20. Simplify $49^{\frac{1}{2}} - 27^{\frac{1}{3}}$.

20–1 10

20–2 -2

20–3 $15\frac{1}{2}$

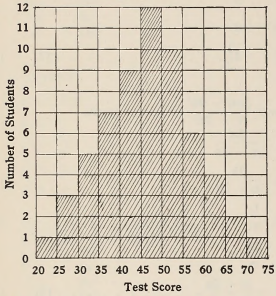
20–4 4

20–5 $22\frac{1}{6}$ . . . . . 20( )

Go on to the next part.

PART II

Directions: Continue as in the preceding part.



1. The graph above shows that the largest number of students have scores in the interval

   1–1 20–25
   1–2 40–45
   1–3 45–50
   1–4 50–55
   1–5 70–75 . . . . . .1( )

2. Charles paid 20¢, 30¢, 25¢, and 35¢ for lunches Monday through Thursday in the school cafeteria. What did he pay for lunch on Friday if his average for the week was 25¢?

   2–1 15¢
   2–2 20¢
   2–3 25¢
   2–4 27¢
   2–5 30¢ . . . . . .2( )

3. The number of people present at a baseball game if the receipts were **d** dollars and each person paid **c** dollars admission was

   3–1 cd
   3–2 d–c
   3–3 c–d
   3–4 $\frac{d}{c}$
   3–5 $\frac{c}{d}$ . . . . . .3( )

4. Tom cut **x** yards of cord into pieces one foot long. How many pieces should he have?

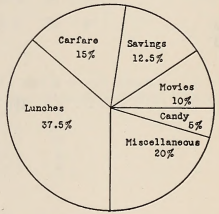
   4–1 $\frac{x}{3}$
   4–2 3x
   4–3 x
   4–4 $\frac{x}{12}$
   4–5 12x . . . . . .4( )

5. The interest on **x** dollars for one year at 3% simple interest is

   5–1 $\frac{x}{.03}$ dollars
   5–2 x − .03 dollars
   5–3 1.03x dollars
   5–4 x + .03 dollars
   5–5 .03x dollars . . . . . .5( )

6. If Robert had $6 more he would have as much money as John. Together they have $80. How much money has Robert?

   6–1 $37
   6–2 $40
   6–3 $43
   6–4 $74
   6–5 $86 . . . . . .6( )



7. Dan earns $4 each week. If he spends it as indicated in the circle graph above, how much money does he save each week?

   7–1 67¢
   7–2 50¢
   7–3 40¢
   7–4 $12\frac{1}{2}$¢
   7–5 5¢ . . . . . .7( )

Go on to the next page.

8. Central High School scored **24** points in a football game. If they made **n** points in the first quarter, twice as many points in the second quarter, the same number of points in the third quarter as in the first, but failed to score in the last quarter, how many points did they score in the second quarter?

8–1 **6**
8–2 **8**
8–3 **12**
8–4 **16**
8–5 **18** . . . . . .8( )

9. Three newsboys challenge one another to a ten-minute paper-selling contest. John sells **4** fewer papers than Harry, and Bill sells twice as many as John. All three together sell **108** papers. If Harry sells **x** papers, which equation would you use to find **x**?

9–1 $3x - 8 = 108$
9–2 $3x - 12 = 108$
9–3 $4x - 8 = 108$
9–4 $4x + 4 = 108$
9–5 $4x - 12 = 108$ . . . . . .9( )

10. The length of a given rectangle is **4** times its width. If **w** is the width in inches, what is the area of the rectangle?

10–1 w^4 **sq in.**
10–2 **10w sq in.**
10–3 $4w^2$ **sq in.**
10–4 **4w sq in.**
10–5 **5w sq in.** . . . . . .10( )

11. A railway ticket for a journey of **60 mi** costs **x** dollars. If the cost is directly proportional to the distance traveled, what would you pay for a ticket for a trip of **200 mi**?

11–1 $\frac{200x}{60}$ **dollars**
11–2 $\frac{60(200)}{x}$ **dollars**
11–3 **60(200)x dollars**
11–4 $\frac{200}{60x}$ **dollars**
11–5 $\frac{60x}{200}$ **dollars** . . . . . .11( )

x	2	4	6	8	10
y	1	3	5	7	9

12. The table above gives values of **y** corresponding to values of **x**. The equation showing the relationship between **x** and **y** is

12–1 $y = \frac{x}{2}$
12–2 $y = 2x$
12–3 $y = x + 1$
12–4 $y = x - 1$
12–5 $y = x + 2$ . . . . . .12( )

13. If a taxicab fare is **25¢** for the first mile and **20¢** for each additional mile, then **c**, the cost in cents for **n** miles, is represented by

13–1 $c = n(20 + 25)$
13–2 $c = 25 + 20n$
13–3 $c = (n - 1)(20 + 25)$
13–4 $c = 25 + 20(n - 1)$
13–5 $c = 25 + \frac{n}{20}$ . . . . . .13( )

14. The formula for the area of a rug to be used in a room ***l*** **ft** long and **w ft** wide, if a border **2 ft** wide is to be left all around the room, is

14–1 $A = lw - 2$
14–2 $A = lw + 4$
14–3 $A = lw - 4$
14–4 $A = (l - 2)(w - 2)$
14–5 $A = (l - 4)(w - 4)$ . . . . .14( )

15. My car cost **25%** more than Mr. Smith's. If **I** paid **x** dollars for mine, what did Mr. Smith pay for his?

15–1 $\frac{100x}{125}$
15–2 $\frac{125x}{100}$
15–3 **.25x**
15–4 $x + .25x$
15–5 $x - .25x$ . . . . . .15( )

Go on to the next part.

PART III

Directions: Continue as in the preceding part.

1. d multiplied by d equals

1–1 1

1–2 $2d$

1–3 d^2

1–4 $\frac{d}{d}$

1–5 $\frac{d}{2}$ . . . . . 1( )

2. The fourth power of r is

2–1 $\frac{r}{4}$

2–2 r^4

2–3 $4r$

2–4 $\sqrt[4]{r}$

2–5 r^4 . . . . . 2( )

3. $5c + 3 - (-3c) - 4c$ equals

3–1 $12c + 3$

3–2 $-2c + 3$

3–3 $-8c$

3–4 $4c + 3$

3–5 $10c$ . . . . . 3( )

4. The number of weeks in r days is

4–1 $7r$

4–2 $52r$

4–3 $\frac{7}{r}$

4–4 $\frac{r}{52}$

4–5 $\frac{r}{7}$ . . . . . 4( )

5. If $\frac{s}{w} = t$, then w equals

5–1 $\frac{s}{t}$

5–2 $t - \frac{1}{s}$

5–3 st

5–4 $\frac{1}{t} - \frac{1}{s}$

5–5 $\frac{t}{s}$ . . . . . 5( )

6. $\frac{12k^5 - 15k}{-3k}$ equals

6–1 $-4k^4 + 5k$

6–2 $-4k^4 + 5$

6–3 $4k^4 - 5$

6–4 $-4k^4 - 15k$

6–5 $12k^5 + 5$ . . . . . 6( )

7. If $\frac{c}{2} = \frac{n}{b}$, then b equals

7–1 $\frac{2}{c} - \frac{1}{n}$

7–2 $\frac{c}{2n}$

7–3 $\frac{n - c}{2}$

7–4 $\frac{c}{2} - n$

7–5 $\frac{2n}{c}$ . . . . . 7( )

8. Which one of the following is true?

8–1 $a^2 + b^2 = (a + b)(a + b)$

8–2 $(a - b)^2 = (b - a)^2$

8–3 $\frac{a^2 - b^2}{a - b} = a - b$

8–4 $(a - b)^2 = (a + b)(a - b)$

8–5 $\frac{(a - b)^2}{a + b} = a - b$ . . . . . 8( )

9. If $px + e = v$, then x equals

9–1 $p(v - e)$

9–2 $v - \frac{e}{p}$

9–3 $\frac{v}{p} - e$

9–4 $\frac{v - e}{p}$

9–5 $v - e - p$ . . . . . 9( )

10. $\frac{\sqrt{21}}{\sqrt{7}}$ equals

10–1 $\sqrt{3}$

10–2 $3\sqrt{7}$

10–3 3

10–4 $7\sqrt{3}$

10–5 $\sqrt{14}$ . . . . . 10( )

Go on to the next page.

11. The volume of a pyramid is one-third the area of the base times the altitude, $V = \frac{Ba}{3}$. If the volume of a pyramid is 14 and the area of the base is **x**, the altitude is

11–1 $\frac{x}{42}$

11–2 $\frac{3x}{14}$

11–3 $\frac{42}{x}$

11–4 $\frac{14x}{3}$

11–5 $\frac{3}{14x}$ . . . . . 11( )

12. $\frac{5c+3}{2} - \frac{4c-5}{3}$ equals

12–1 $\frac{7c-1}{6}$

12–2 $\frac{7c+19}{6}$

12–3 $\frac{23c+19}{6}$

12–4 $\frac{-2c-9}{6}$

12–5 $\frac{c-2}{5}$ . . . . . 12( )

13. If $\frac{6}{x} = \frac{x}{4}$, one value of **x** is

13–1 $\frac{2}{3}$

13–2 $1\frac{1}{2}$

13–3 $2\sqrt{6}$

13–4 -4

13–5 24 . . . . . 13( )

14. $\frac{3}{r} + \frac{8}{z}$ equals

14–1 $\frac{3r+8z}{rz}$

14–2 $\frac{3r+8z}{r+z}$

14–3 $\frac{3z+8r}{rz}$

14–4 $\frac{3z+8r}{r+z}$

14–5 $\frac{11}{r+z}$ . . . . . 14( )

15. How many baskets, holding **k** apples each, can be filled from a barrel containing **c** apples?

15–1 $\frac{c}{k}$

15–2 $k - c$

15–3 ck

15–4 $c - k$

15–5 $\frac{k}{c}$ . . . . . 15( )

16. "When **x** is divided by **y**, the quotient is **a** and the remainder is **b** + **c**," may be expressed algebraically as

16–1 $\frac{x}{y} = \frac{a}{y} + b + c$

16–2 $\frac{x}{y} = a + \frac{b+c}{y}$

16–3 $\frac{x}{y} = a + b + c$

16–4 $\frac{x}{y} = a + y(b + c)$

16–5 $\frac{x}{y} = ay + b + c$ . . . . . 16( )

17. If $r^2 = 9d$, **r** equals

17–1 $\pm 9\sqrt{d}$

17–2 $\pm 3d$

17–3 $\pm 3\sqrt{d}$

17–4 $\pm 9d$

17–5 $\pm \sqrt{3d}$ . . . . . 17( )

18. If $Rx = px - 5$, then **x** equals

18–1 $\frac{R}{p-5}$

18–2 $\frac{-5}{R-p}$

18–3 $\frac{-5}{Rp}$

18–4 $\frac{R-p}{5}$

18–5 $R - \frac{5}{p}$ . . . . . 18( )

19. Ben is paying off a debt of **D** dollars in monthly installments of **d** dollars each. He has made **x** payments. Which formula tells what sum, **S**, he still has to pay?

19–1 $S = \frac{D}{x} - d$

19–2 $S = x(D - d)$

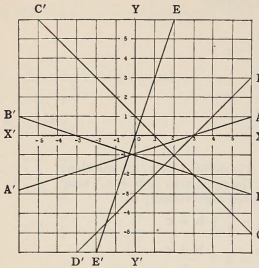
19–3 $S = D - d - x$

19–4 $S = D - xd$

19–5 $S = x\left(\frac{D}{d}\right)$ . . . . . 19( )

Go on to the next page.

Questions **20** through **24** refer to the graph below.



20. For the line **DD′**, when $y = -2$, **x** equals

20–1 **1**
20–2 -2
20–3 **0**
20–4 **4**
20–5 -5 . . . . . 20( )

21. $x - y = 3$ is the equation of line

21–1 **AA′**
21–2 **BB′**
21–3 **CC′**
21–4 **DD′**
21–5 **EE′** . . . . . 21( )

22. $3x - y = 0$ is the equation of line

22–1 **AA′**
22–2 **BB′**
22–3 **CC′**
22–4 **DD′**
22–5 **EE′** . . . . . 22( )

23. $x + 3y = -3$ is the equation of line

23–1 **AA′**
23–2 **BB′**
23–3 **CC′**
23–4 **DD′**
23–5 **EE′** . . . . . 23( )

24. The solution of the pair of equations represented by lines **BB′** and **CC′** is

24–1 $x = -3, y = -2$
24–2 $x = -3, y = 2$
24–3 $x = -2, y = 3$
24–4 $x = 3, y = -2$
24–5 $x = 3, y = 2$ . . . . . 24( )

* * * * *

25. If $\frac{1}{a} = \frac{1}{n} + k$, then **a** equals

25–1 $\frac{n}{1 + nk}$
25–2 $n - \frac{1}{k}$
25–3 $n - k$
25–4 $n + \frac{1}{k}$
25–5 $\frac{1 + nk}{n}$ . . . . . 25( )

26. The product of $c^{1/2}$ and c^2 is

26–1 **c**
26–2 $c^{3/2}$
26–3 $c^{-3/2}$
26–4 $c^{5/2}$
26–5 c^4 . . . . . 26( )

27. The value of the fraction $\frac{100}{x + 5}$

27–1 is doubled if **x** is doubled.
27–2 is $\frac{1}{3}$ as great if **x** is divided by **3**.
27–3 is squared if **x** is squared.
27–4 decreases when **x** increases.
27–5 decreases when **x** decreases. . . 27( )

28. Which one of the following is true?

28–1 $(n^a)^2 = n^{a+2}$
28–2 $\frac{n^2}{\sqrt{n}} = n^2\sqrt{n}$
28–3 $(-n)^4 = -(n)^4$
28–4 $(n^2)^4 = n^7$
28–5 $n^{-4} = (-n)^4$ . . . . . 28( )

Number wrong	0 | 2	3 | 6	7 | 10	11 | 14	15 | 18	19 | 22	23 | 26	27 | 30	31 | 34	35 | 38	39 | 42	43 | 46	47 | +
Amount to be subtracted	0	1	2	3	4	5	6	7	8	9	10	11	12

Number right_____

Subtract _____
(See table above)

Raw Score = Difference _____

Scaled Score _____
(See table on key)

CALIFORNIA SHORT-FORM TEST OF MENTAL MATURITY

ADVANCED S-FORM

Devised by Elizabeth T. Sullivan, Willis W. Clark, and Ernest W. Tiegs

INSTRUCTIONS TO EXAMINER

When the California Short-Form Test of Mental Maturity is given by use of the special test scoring machine answer sheet, the following directions should be followed:

1. Provide each student to be tested with a copy of the Test Booklet and a copy of the Machine Scoring Answer Sheet (No. B 313).

2. Students should be provided with a suitable pencil approved for this purpose.

3. Students should be instructed not to fold or crease the answer sheet.

4. The answer sheet is numbered to agree with the sections of the test, and it is important that marks on the various sections of the answer sheet agree with the same sections of the test.

5. Students are not to mark on the test booklet in any way, but are to indicate their answers on the special answer sheet.

6. Have students record their names, other identifying data, and the form of the test on the answer sheet.

7. Read the directions printed on the answer sheet aloud. Be sure that students understand the method of marking the correct responses as shown in the sample. Note that in Test 2 and 3, the examiner is to tell the students the correct answer as given in the directions.

8. If students have not previously used machine scoring answer sheets, it may be desirable to illustrate on the blackboard or give considerable explanation.

9. Emphasize that no writing is to be done on the test booklet and that the pencil line which indicates the student's choice of correct answers must be a heavy black line.

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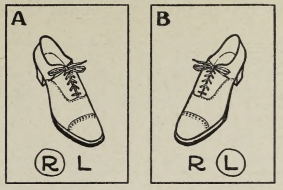
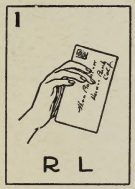
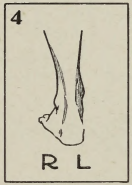
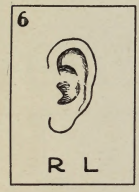
TEST 1.

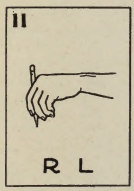
Directions: The objects on this page are either rights or lefts. Mark on the answer sheet under the R for rights, and under the L for lefts. A and B are correctly marked on the answer sheet.

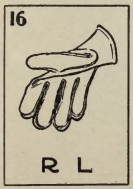


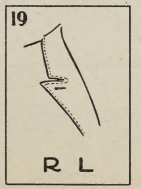






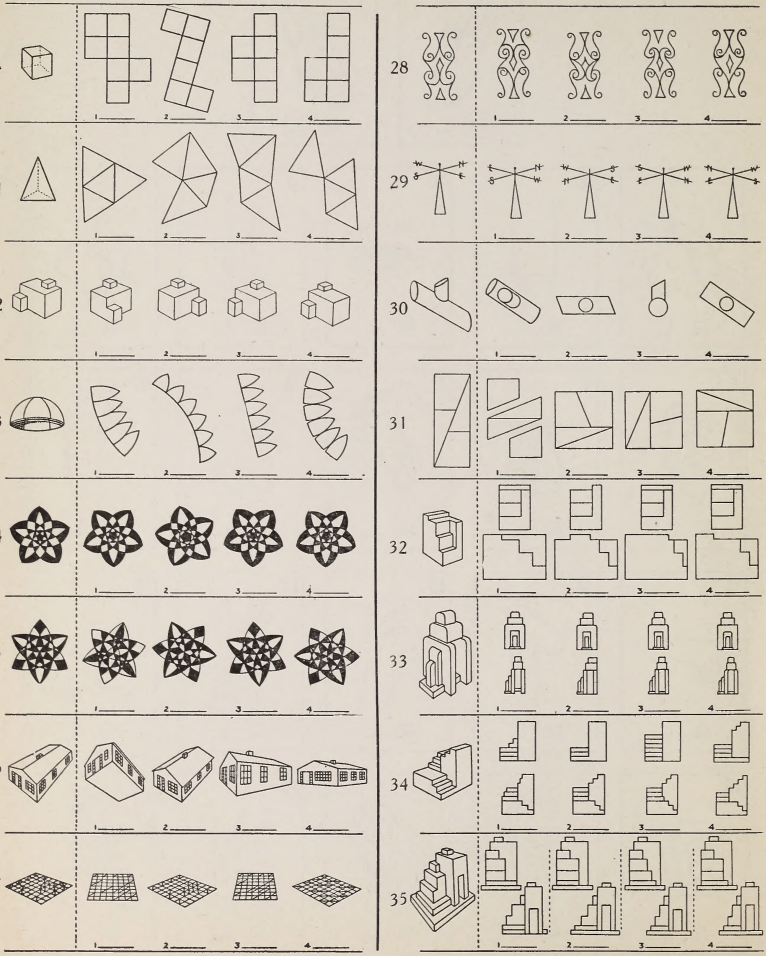






TEST 2.

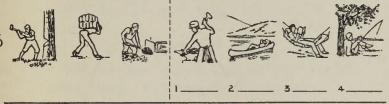
Directions: In each row find a drawing that is either the same drawing or different views of the first drawing. Mark the number of the drawing which you have decided is correct on the answer sheet. In the first row the correct answer is 2.

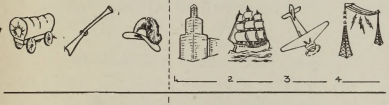


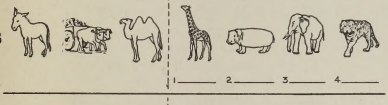
TEST 3.

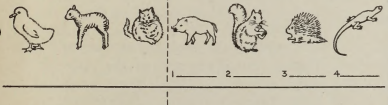
Directions: The first three objects in each row are alike in some way. Find another object in the same row that belongs with them. Mark the number of the drawing which you have decided is correct on the answer sheet. In the first row the correct answer is 3.

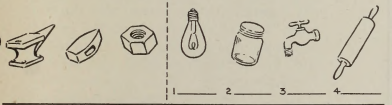
A

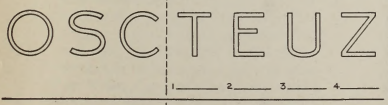
36 

37 

38 

39 

40 

41 

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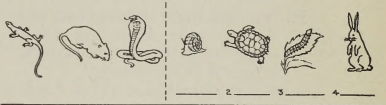
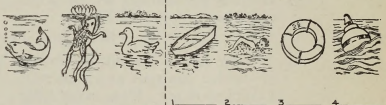
45 

46 

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48

49

50

TEST 4.

Directions: Work these problems on a blank sheet of paper. Mark the letter of the answer on the answer sheet. The correct answer for the first problem is b.

A. If a man earned $25.00 and spent $10.00, how much money did he have left?
Ans.: a $5 b $15 c $20 d $10

51. How many picture postal cards can you buy for 15 cents at the rate of 3 for 5 cents?
Ans.: a 9 b 3 c 15 d 34

52. How many feet of railroad track can be laid with 750 ties if 25 ties are needed for each 50 feet?
Ans.: a 1250 b 1500 c 325 d 30

53. What number if multiplied by 3, is 2 times 9?
Ans.: a 3 b 9 c 18 d 6

54. A sample rug is 12 inches long and 9 inches wide. How long will a larger rug of the same proportions be if it is 36 inches wide?
Ans.: a 108 in. b 48 in. c 15 in. d 36 in.

55. What is the number which if divided by 4, is 1/6 of 72?
Ans.: a 12 b 18 c 48 d 3

56. A high school student borrowed $75.00 for one year at 6% to start a chicken ranch. How many little chickens must he sell at 10 cents each to pay back the money he borrowed with interest?
Ans.: a 45 b 450 c 750 d 795

57. A dealer allowed an old customer a discount of 10% on the marked price of book cases. What is the marked price of a book case for which this customer paid him $36.00?
Ans.: a $40 b $32.40 c $3.60 d $39.60

58. A circular flower bed 7 feet in diameter is to be bordered by plants set one foot apart. What will be the cost of the plants at the rate of 2 for 15 cents? (Circumference of a circle is about 3⅐ times the diameter.)
Ans.: a 52¢ b $1.65 c 70¢ d $1.57½

59. A man placed four stepping stones one foot square in a row in a section of his garden so that there were equal spaces on all four sides of each of the stones. If the section was 3 feet wide, how long was it?
Ans.: a 12 ft. b 3 ft. c 9 ft. d 8 ft.

60. Ben lives 1.5 miles east of the library. James lives 2.5 miles directly west of the library. On a scale of ½ inch = 1 quarter mile, how many inches will represent the distance between the boys' houses?
Ans.: a 8in. b 16 in. c 6 in. d 2 in.

61. What is the number which if added to 5 is 3 less than 1/3 of 3/5 of 60?
Ans.: a ½ b 9 c 4 d 12

62. A gallon of water weighs 8.4 pounds. A gallon of gasoline weighs 68 per cent as much as a gallon of water. A pilot flying the air mail carried 50 gallons. How many pounds did this gasoline weigh?
Ans.: a 285 b 285.6 c 278.6 d 380

63. A coffee shop buys a blend of coffee composed of 2/3 of Grade A at 60 cents a pound and 1/3 of Grade B at 30 cents a pound. If they change the mixture, using 1/3 of Grade A and 2/3 of Grade B, how much will they save on every 10 pounds of coffee?
Ans.: a 3¢ b 10¢ c 30¢ d $1.00

64. A man's will provides that his estate of $15,000.00 should be divided as follows: 2/5 to his wife and 1/5 each to three children, except that in the event any of the children were deceased, their share should be divided equally between the remaining children and the wife. Two children were killed in an automobile accident. How much did the remaining child receive from the estate?
Ans.: a 1/5 b $6000.00 c $4500 d $5000

65. If a set of tires for one automobile costs one-half of what a set costs for another automobile; and if three sets of the cheaper tires last only as long as two sets of the more expensive kind, the total cost of the cheaper tires during a given period will average what fraction or per cent of the cost of the more expensive kind?
Ans.: a 1/3 or 33 1/3% b ½ or 50%
c ¾ or 75% d 1½%

TEST 5.

Directions: Read each group of statements and mark the number of the correct logical answer on the answer sheet. The correct answer for the first problem is 3.

0. All four-footed creatures are animals.
All horses are four-footed. Therefore

1 Creatures other than horses can walk
2 All horses can walk
3 All horses are animals

66. Elm Street is parallel to Oak Street.
Oak Street is parallel to Palm Avenue.
Therefore
1 Elm Street crosses Palm Avenue
2 Palm Avenue is longer than Elm Street
3 Elm Street is parallel to Palm Avenue

67. George Washington was a skillful general.
George Washington was President of the United States.
Therefore

1 Skillful generals make good presidents
2 A President of the United States was a skillful general
3 Good presidents make skillful generals

68. If he steers toward the land he will be wrecked; and if he steers toward the open sea he will be wrecked; but, he must steer either toward the land or toward the open sea. Therefore

1 He should head for the open sea
2 The coast is dangerous for ships
3 He will be wrecked

69. If the wind changes it will either grow warmer or it will storm.
The wind does not change. Therefore

1 It will probably grow warmer
2 The conclusion is uncertain
3 It will not grow warmer nor will it storm

70. X is younger than Y.
Y is younger than Z. Therefore

1 Y is younger than X
2 X is younger than Z
3 Y has lived longer than Z

71. All circles are round figures.
The figure is not round. Therefore

1 It is oval
2 It is either a square or a triangle
3 It is not a circle

72. A is situated to the east of B.
B is situated to the east of C. Therefore

1 C is situated close to A
2 A is situated to the east of C
3 C is nearer to A than to B

73. If he is to complete his high school course, he must avoid wasting his energy and his money.
But he will not avoid wasting his energy, or he will not avoid wasting his money. Therefore

1 He will not complete his high school course
2 He will be sorry some day
3 He should be criticized for not doing better

74. If the students are in error, your refusal to listen to their side is unreasonable;
If they are not in error, your refusal is unjust.
But, the students are in error or they are not. Therefore

1 Your refusal is justifiable
2 Your refusal is either unreasonable or it is unjust
3 Your refusal may be reconsidered later

75. Three boys are up on a ladder.
Tom is farther up the ladder than Paul.
Jim is farther up than Tom.
Which boy is in the middle position on the ladder?

1 Tom 2 Paul 3 Jim

76. A is either B or C or D.
A is not B. Therefore

1 A is C 2 A is either C or D
3 The conclusion is uncertain

77. If he were loyal he would not speak unkindly of his family in earnest.
If he were wise he would not speak unkindly of them in jest.
Either he speaks unkindly in earnest or in jest.
Therefore

1 He is either not loyal or not wise
2 He is unkind
3 The conclusion is uncertain

78. If A is B, E is F; if C is D, G is H.
Either A is B or C is D. Therefore

1 A is F or C is H
2 Either E is F or G is H
3 The conclusion is uncertain

79. A is between B and C.
B is between C and D. Therefore

1 A is not between C and D
2 A is between B and D
3 A is nearer to B than to D

80. If A is B, E is F, and if A is B, G is H
Either E is not F or G is not H. Therefore

1 A is not B 2 A is B
3 The conclusion is uncertain

Directions: Mark on the answer sheet the number of the word which means the same or about the same as the first word. The answer for the first one is 3.

0. **blossom** [1] tree [2] vine [3] flower [4] garden

81. **inefficient** [1] avoidable [2] quarrelsome [3] incompetent [4] unruly

82. **confiscate** [1] assert [2] seize [3] compile [4] comfort

83. **malign** [1] insure [2] muffle [3] slander [4] invade

84. **whimsical** [1] accurate [2] fashionable [3] weighty [4] fanciful

85. **avarice** [1] virtue [2] prominence [3] greed [4] honor

86. **eradicate** [1] destroy [2] vacate [3] use [4] solve

87. **impeachment** [1] precedent [2] settlement [3] resignation [4] accusation

88. **discordant** [1] clashing [2] despondent [3] unsteady [4] distinctive

89. **titanic** [1] reddish [2] acid [3] large [4] ancient

90. **edict** [1] decree [2] diction [3] sovereign [4] edition

91. **recumbent** [1] cumbersome [2] curved [3] reclining [4] saving

92. **caprice** [1] action [2] whim [3] capture [4] tact

93. **expedite** [1] expel [2] dictate [3] delay [4] hasten

94. **loquacious** [1] talkative [2] logical [3] legal [4] delicious

95. **idiosyncracy** [1] irritability [2] peculiarity [3] office [4] imbecility

96. **perfidious** [1] treacherous [2] fragrant [3] studious [4] responsible

97. **artifice** [1] artless [2] hate [3] definition [4] device

98. **anomaly** [1] ceremony [2] illness [3] irregularity [4] normal

99. **reciprocal** [1] charming [2] mutual [3] agreeable [4] meditative

100. **travesty** [1] burlesque [2] tragedy [3] meeting [4] hotel

101. **obtuse** [1] pointed [2] reversible [3] blunt [4] objectionable

102. **abstemious** [1] stormy [2] excessive [3] mournful [4] temperate

103. **tangent** [1] blend [2] agent [3] touching [4] sensing

104. **extraneous** [1] extra [2] foreign [3] transparent [4] noisy

105. **erudite** [1] crude [2] learned [3] rugged [4] polite

106. **ameliorate** [1] improve [2] harden [3] dilute [4] decorate

107. **malapert** [1] sick [2] lazy [3] slow [4] saucy

108. **opulence** [1] jewel [2] generosity [3] wealth [4] honor

109. **urbanity** [1] loyalty [2] refinement [3] weakness [4] barbarism

110. **propinquity** [1] nearness [2] curiosity [3] diligence [4] propriety

111. **trajectory** [1] court [2] project [3] area [4] curve

112. **corollary** [1] crown [2] inference [3] enclosure [4] supersede

113. **ostensible** [1] actual [2] available [3] genuine [4] pretended

114. **salient** [1] salty [2] outstanding [3] merciful [4] agreeable

115. **probity** [1] uprightness [2] interference [3] suspicion [4] weight

116. **acephalous** [1] false [2] warlike [3] headless [4] sensible

117. **porphyry** [1] papyrus [2] rock [3] cave [4] manuscript

118. **strident** [1] muscular [2] shrill [3] battered [4] strong

119. **effete** [1] exhausted [2] festive [3] fragile [4] plentiful

120. **tyro** [1] scold [2] village [3] law [4] beginner

121. **perimeter** [1] measure [2] instrument [3] boundary [4] difficulty

122. **diurnal** [1] seasonable [2] occasional [3] timely [4] daily

123. **obloquy** [1] disaster [2] blame [3] pride [4] obligation

124. **eyot** [1] island [2] lake [3] river [4] insect

125. **detritus** [1] fossil [2] dextrous [3] fragment [4] poem

126. **palladium** [1] burden [2] safeguard [3] title [4] residence

127. **quiddity** [1] oddity [2] doubt [3] essence [4] presence

128. **ambient** [1] uncertain [2] surrounding [3] surprising [4] well-wishing

129. **orrery** [1] book [2] prophecy [3] apparatus [4] error

130. **syzygy** [1] separation [2] choice [3] conjunction [4] nonsense

AMERICAN COUNCIL ON EDUCATION

COÖPERATIVE GENERAL MATHEMATICS TEST

FOR HIGH SCHOOL CLASSES

REVISED SERIES FORM O

by
H. T. LUNDHOLM, State University of Iowa
and
L. P. SICELOFF, Columbia University

Please print:

Name.....Date.....
Last First Middle

Grade or Class.....Age.....Date of Birth.....
Yrs. Mos.

School.....City.....Sex.....
M. or F.

Instructor.....

Number of years (one semester = ½ year; one quarter = ⅓ year) you have studied:

Algebra.....Plane Geometry.....Solid Geometry.....Trigonometry.....

General Directions: Do not turn this page until the examiner tells you to do so. This examination consists of three parts, and requires 40 minutes of working time. The directions for each part are printed at the beginning of the part. Read them carefully, and proceed at once to answer the questions. DO NOT SPEND TOO MUCH TIME ON ANY ONE ITEM; ANSWER THE EASIER QUESTIONS FIRST; then return to the harder ones, if you have time. There is a time limit for each part. You are not expected to answer all the questions in any part in the time limit; but if you should, go on to the next part. If you have not finished Part I when the time is up, stop work on that part and proceed at once to Part II. No questions may be asked after the examination has begun.

By exercising careful judgment and making shrewd guesses you may profitably answer questions about which you are not absolutely sure; but since your score will be the number of correct answers diminished by a number proportional to the number of wrong answers, you should avoid answering questions about which you are totally ignorant. Shrewd guessing based on intelligent inference will improve your score, but wild guessing on questions that are entirely unknown to you will waste time which you could better put on other questions in the test, and may result in a large subtraction from the number of your correct answers.

Part	I	II	III	Total
Minutes	15	10	15	40

Raw Score	Scaled Score (See table on key)	Percentile

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15 Amsterdam Avenue, New York City

PART I

Directions: Each problem below is accompanied by several possible answers, only one of which is correct. In each problem find which is the correct answer, then write the **number** of that correct answer in the parentheses to the right of the problem.

1. What number is ten times as large as **3.6**?
   1–1 **.36**
   1–2 **3.60**
   1–3 **30.6**
   1–4 **360**
   1–5 **36** . . . . . .1( )

2. What is the product of $2\frac{1}{3}$ and **3**?
   2–1 $5\frac{1}{3}$
   2–2 **6**
   2–3 $6\frac{1}{3}$
   2–4 **7**
   2–5 $\frac{12}{3}$ . . . . . .2( )

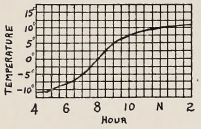

Figure 1

3. In the temperature graph (Figure 1), how many degrees did the temperature rise between 5 o'clock and noon?
   3–1 **10°**
   3–2 **20°**
   3–3 **15°**
   3–4 **25°**
   3–5 **22°** . . . . . .3( )

4. What fractional part of a yard is **2** feet and **3** inches?
   4–1 $\frac{2}{3}$
   4–2 $\frac{5}{6}$
   4–3 $\frac{3}{4}$
   4–4 $\frac{7}{9}$
   4–5 $\frac{1}{4}$ . . . . . .4( )

5. What per cent of **2** is $\frac{1}{2}$?
   5–1 **400%**
   5–2 **20%**
   5–3 **75%**
   5–4 **40%**
   5–5 **25%** . . . . . .5( )

6. What is the value of the expression $(-3a^3)$ if **a** is (-2)?
   6–1 **−24**
   6–2 **24**
   6–3 **18**
   6–4 **−18**
   6–5 **−8** . . . . . .6( )

7. How many degrees are there in the angle formed by the bisectors of two adjacent supplementary angles?
   7–1 **120°**
   7–2 **90°**
   7–3 **30°**
   7–4 **45°**
   7–5 **60°** . . . . . .7( )

8. In a triangle **ABC**, angle **A** is **34°** and angle **B** is **62°15′**. How large is angle **C**?
   8–1 **84°15′**
   8–2 **83°15′**
   8–3 **27°45′**
   8–4 **53°45′**
   8–5 **83°45′** . . . . . .8( )

9. What is the value of **a** in the formula $S = \frac{a}{1 - r}$ if $S = 2$ and $r = -\frac{1}{2}$?
   9–1 **1**
   9–2 **2**
   9–3 **3**
   9–4 **4**
   9–5 **2½** . . . . . .9( )

10. How much is **½%** of **$160.00**?
    10–1 **$80.00**
    10–2 **$1.60**
    10–3 **$8.00**
    10–4 **$0.80**
    10–5 **$16.00** . . . . . .10( )

11. By how much does $3x + 2y$ exceed $x - y$?
    11–1 $2x + y$
    11–2 $4x + y$
    11–3 $4x + 3y$
    11–4 $2x + 3y$
    11–5 $2x + 2y$ . . . . . .11( )

12. How many degrees are there in each base angle of an isosceles triangle if the exterior angle at its vertex is **130°**?
    12–1 **65°**
    12–2 **25°**
    12–3 **30°**
    12–4 **60°**
    12–5 **50°** . . . . . .12( )

Go on to the next page.

13. What is the divisor if the dividend is $3a^3$ and the quotient is $\frac{a}{2}$?

13–1 $\frac{3a^4}{2}$
13–2 2
13–3 $\frac{1}{6a^2}$
13–4 $6a^2$
13–5 $\frac{1}{2a^2}$ . . . . . .13( )

14. $V = \pi r^2 h$ is a formula for the volume of a cylinder. If $V = 44$, $h = 3\frac{1}{2}$, and π is taken as $\frac{22}{7}$, the radius, r, is

14–1 $\frac{2}{7}$
14–2 2
14–3 $\sqrt{44}$
14–4 4
14–5 22 . . . . . .14( )

15. The width of a certain rectangle is W and its perimeter is 14. The algebraic expression for the length of the rectangle is

15–1 $7 - W$
15–2 $W - 7$
15–3 $\frac{7 - W}{2}$
15–4 $2(W - 7)$
15–5 $2W$ . . . . . .15( )

16. A line segment 32 inches long is divided internally into two segments whose ratio is $\frac{5}{3}$. The length of the longer segment is

16–1 12.8 inches.
16–2 20 inches.
16–3 12 inches.
16–4 28 inches.
16–5 22 inches. . . . . .16( )

17. How many degrees are there in an angle which is one-fourth of its complement?

17–1 $18°$
17–2 $22°30'$
17–3 $30°$
17–4 $45°$
17–5 $36°$ . . . . . .17( )

18. A storekeeper fixes the selling price of an article on the following basis:

Purchase price	65%
Overhead	20%
Profit	15%
Selling price	100%

In making a circle graph of these data, how many degrees are there in the central angle of the sector representing the overhead?

18–1 $200°$
18–2 $20°$
18–3 $144°$
18–4 $36°$
18–5 $72°$ . . . . . .18( )

19. If one inch is added to the altitude of a certain parallelogram, its area is increased by 8 square inches. What is the length of the base of the parallelogram?

19–1 16 inches.
19–2 $\sqrt{8}$ inches.
19–3 $7\frac{1}{2}$ inches.
19–4 64 inches.
19–5 8 inches. . . . . .19( )

20. Let A represent the average of two numbers x and y. The formula for x in terms of A and y is

20–1 $2A + y$
20–2 $2A - y$
20–3 $y - 2A$
20–4 $\frac{A - y}{2}$
20–5 $\frac{A + y}{2}$ . . . . . .20( )

21. What is the numerical value of the expression $8^{4/3}$?

21–1 $10\frac{2}{3}$
21–2 $1385\frac{1}{3}$
21–3 12
21–4 $\sqrt[3]{32}$
21–5 16 . . . . . .21( )

Go on to the next page.

Items 22 and 23 refer to Figure 2.

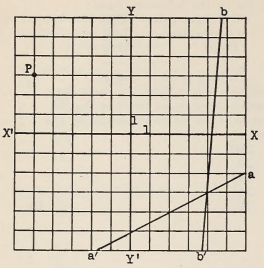

Figure 2

22. What is the value of the ordinate of point **P**?

22–1 **6**
22–2 **−6**
22–3 **3**
22–4 **−3**
22–5 **9** . . . . . **.22(** **)**

23. If the equations whose graphs are lines **aa′** and **bb′** were solved simultaneously, what would be their common **x** value?

23–1 **10**
23–2 **−4**
23–3 **3**
23–4 **4**
23–5 **−3** . . . . . **.23(** **)**

24. How many degrees are there in an acute angle whose sine equals $\frac{1}{2}$?

24–1 **60°**
24–2 **72°**
24–3 **30°**
24–4 **45°**
24–5 **50°** . . . . . **.24(** **)**

25. The lateral faces of a regular pyramid are congruent isosceles triangles. By what must the area of one of these triangles be multiplied in finding the lateral area of a regular pyramid of **n** sides?

25–1 **altitude**
25–2 **base**
25–3 **perimeter**
25–4 **4**
25–5 **n** . . . . . **.25(** **)**

26. For what two positive values of **y** is the expression $y^4 - 5y^2 + 4$ equal to zero?

26–1 **5, 3**
26–2 **1, 4**
26–3 **2, 4**
26–4 **1, 2**
26–5 **3, 6** . . . . . **.26(** **)**

27. The value of **N** in the equation $4\sqrt[3]{16} - \sqrt[3]{54} = N\sqrt[3]{2}$ is

27–1 $\sqrt[3]{5}$

27–2 $-4\sqrt[3]{18}$

27–3 $\frac{5}{3}$

27–4 $\frac{8}{\sqrt[3]{2}} - 3$

27–5 **5** . . . . . **.27(** **)**

28. The total surface area of a rectangular parallelopiped whose width, length, and height are **2, 3,** and **5** inches, respectively, is

28–1 **10** square inches.
28–2 **62** square inches.
28–3 **30** square inches.
28–4 **31** square inches.
28–5 **25** square inches. . . . . **.28(** **)**

29. In which quadrant (**I, II, III,** or **IV**) will the terminal side of an angle lie if its tangent is positive and its sine is negative?

29–1 **I**
29–2 **II**
29–3 **III**
29–4 **IV**
29–5 **Impossible case** . . . . . **.29(** **)**

30. In a right triangle the tangent of one acute angle is $\frac{3}{4}$. What is the cosine of the other acute angle?

30–1 $\frac{4}{3}$

30–2 $\frac{3}{4}$

30–3 $\frac{3}{5}$

30–4 $\frac{4}{5}$

30–5 $\frac{5}{3}$ . . . . . **.30(** **)**

Go on to the next page.

PART II

Directions: Proceed as in the preceding exercises.

31. The result obtained in a multiplication problem is called the
31–1 **quotient.**
31–2 **remainder.**
31–3 **sum.**
31–4 **product.**
31–5 **addend.** . . . . . .**31**( )

32. In algebra the number **5** in the expression $5x^2$ is known as
32–1 **an exponent.**
32–2 **an index number.**
32–3 **a coefficient.**
32–4 **a power.**
32–5 **a characteristic.** . . . . . .**32**( )

33. If the sum of two angles is **90°** the angles are
33–1 **adjacent.**
33–2 **complementary.**
33–3 **equal.**
33–4 **supplementary.**
33–5 **vertical.** . . . . . .**33**( )

34. Which one of the following statements is not always true of a parallelogram?
34–1 **The opposite sides are equal.**
34–2 **The diagonals bisect each other.**
34–3 **The opposite sides are parallel.**
34–4 **The opposite angles are equal.**
34–5 **The diagonals are perpendicular to each other.** . . . . . .**34**( )

35. Solve the equation $L = \frac{mt - g}{t}$ for **t.**

35–1 $t = \frac{g}{L - m}$

35–2 $t = \frac{L - g}{m}$

35–3 $t = \frac{g}{m - L}$

35–4 $t = \frac{L + g}{m}$

35–5 $t = \frac{L}{m} + g$ . . . . . .**35**( )

36. Which one of the following angles is measured directly by its intercepted arc of a circle?
36–1 **An angle formed by two secants intersecting at a point without a circle.**
36–2 **An angle formed by a tangent and a chord.**
36–3 **An angle formed by two tangents.**
36–4 **An angle formed by two radii.**
36–5 **An angle formed by two chords intersecting within a circle.** . .**36**( )

37. Which of the following expressions has the largest numerical value if **x** is a positive integer greater than **1** and less than **100**?

37–1 $\frac{x - 1}{x}$

37–2 $\frac{x}{x + 1}$

37–3 $\frac{1}{x}$

37–4 $\frac{x + 1}{x}$

37–5 $\frac{x}{x - 1}$ . . . . . .**37**( )

38. What unit of weight in the metric system is equal approximately to **2.2** pounds?
38–1 **kilogram.**
38–2 **hectogram.**
38–3 **gram.**
38–4 **decigram.**
38–5 **centigram.** . . . . . .**38**( )

39. The product of $\sqrt{6}$ and $\sqrt{8}$ in its simplest radical form is
39–1 $4\sqrt{3}$
39–2 $3\sqrt{2}$
39–3 $\sqrt{14}$
39–4 $2\sqrt{3}$
39–5 $3\sqrt{4}$ . . . . . .**39**( )

40. Which one of the following points lies on the straight line which is the graph of the equation $3x + 2y = -6$?
40–1 **(3, −1)**
40–2 **(−3, 0)**
40–3 **(2, −3)**
40–4 **(0, −3)**
40–5 **(1, −3)** . . . . . .**40**( )

41. A right triangle has two equal acute angles. What is the expression for the area of the triangle if its hypotenuse is **h**?

41–1 $\frac{h}{2}\sqrt{2}$

41–2 $\frac{h^2}{4}$

41–3 $\frac{h^2}{4}\sqrt{3}$

41–4 $\frac{h^2}{8}$

41–5 $\frac{h}{2}\sqrt{3}$ . . . . . .**41**( )

Go on to the next page.

42. Which of the trigonometric functions of an acute angle, in a right triangle, is defined as the ratio of the "side adjacent" to the "side opposite" that angle?
42–1 **tangent.**
42–2 **sine.**
42–3 **secant.**
42–4 **cosine.**
42–5 **cotangent.** . . . . . .42( )

43. What is the cosine of a 45° angle?
43–1 $\frac{\sqrt{3}}{2}$
43–2 $\sqrt{3}$
43–3 $\frac{\sqrt{2}}{2}$
43–4 $\sqrt{2}$
43–5 $\frac{\sqrt{2}}{3}$ . . . . . .43( )

44. What is the total area of a cylinder of revolution whose altitude is 5 inches and radius of the base is 3 inches?
44–1 48π square inches.
44–2 45π square inches.
44–3 36π square inches.
44–4 24π square inches.
44–5 15π square inches. . . . . .44( )

45. Under which one of the following conditions is $a + b + c$ a geometric progression?
45–1 $a = \sqrt{bc}$
45–2 $b = \frac{a + c}{2}$
45–3 $b = \sqrt{ac}$
45–4 $b = \frac{a - c}{2}$
45–5 $c = \sqrt{ab}$ . . . . . .45( )

PART III

Directions: Proceed as in the preceding exercises.

46. Two numbers whose sum is 52 have a ratio 4 : 9. The smaller number is
46–1 12
46–2 16
46–3 13
46–4 4
46–5 36 . . . . . .46( )

47. An investor receives $14 interest annually on an investment of $400. How much interest, annually, will he receive from an investment of $560 if the ratio between the interest and the amount invested remains the same?
47–1 $15.60
47–2 $156
47–3 $196
47–4 $19.60
47–5 $78.40 . . . . . .47( )

48. Two integers are in the ratio of 4 to 3 and their product plus their sum is 62. What is the larger of the two integers?
48–1 31
48–2 14
48–3 10
48–4 4
48–5 8 . . . . . .48( )

49. A publishing house offers a certain magazine at the price of $2.00 per year, which is 20% less than the regular price. What is the regular price of the magazine?
49–1 $2.70
49–2 $2.20
49–3 $1.80
49–4 $1.60
49–5 $2.50 . . . . . .49( )

50. The difference between one-third and one-fifth of a certain number is eleven less than one-half the number. What is the number?
50–1 11
50–2 20
50–3 30
50–4 45
50–5 15 . . . . . .50( )

51. In an isosceles triangle ABC, the vertex angle C is 72 degrees. Altitudes drawn from A and B intersect at a point K. How many degrees are there in angle KAC?
51–1 36°
51–2 54°
51–3 30°
51–4 18°
51–5 72° . . . . . .51( )

52. Let x represent the side of a given square. What algebraic expression will represent the perimeter of a square whose area is four times that of the given square?
52–1 16x
52–2 25x
52–3 64x
52–4 4x
52–5 8x . . . . . .52( )

Go on to the next page.

53. A printing press did a certain amount of work in **5** hours. When a second press was installed an equal amount of work was completed by both presses in **3** hours. How many hours would be required for the second press to complete that amount of work alone?

53–1 **6** hours.
53–2 **7½** hours.
53–3 **10** hours.
53–4 **8** hours.
53–5 **5** hours. . . . . .**53**( )

54. A chemist has **5** ounces of an acid solution of **22%** strength. How many ounces of pure acid must he add in order to have a solution that is of **35%** strength?

54–1 **1** oz.
54–2 **8** oz.
54–3 **3** oz.
54–4 **13** oz.
54–5 **5** oz. . . . . .**54**( )

55. A line **AB**, **10** inches in length, moves in such a manner that each end is in one of two perpendicular lines. The locus of the midpoint of **AB** is a circle. What is the radius of this circle?

55–1 **10** inches.
55–2 **20** inches.
55–3 $\sqrt{10}$ inches.
55–4 $\sqrt{5}$ inches.
55–5 **5** inches. . . . . .**55**( )

56. A rectangular piece of tin is twice as long as it is wide. From each corner a **2** inch square is cut out and the sides turned up so as to form a box whose volume is **60** cubic inches. What is the width of the **box**?

56–1 **7** inches.
56–2 **14** inches.
56–3 **3** inches.
56–4 **4** inches.
56–5 **5** inches. . . . . .**56**( )

57. A formula for the area of a sphere is $A = 4\pi r^2$. What is the radius of a sphere whose area, in square units, is numerically equivalent to the circumference, in linear units, of its great circle?

57–1 $\frac{1}{2}$
57–2 **2**
57–3 $\frac{1}{4}$
57–4 **4**
57–5 **8** . . . . . .**57**( )

58. What is the edge of a cube whose volume in cubic inches is numerically equal to one-half its area in square inches?

58–1 **9** inches.
58–2 $\frac{3}{2}$ inches.
58–3 **3** inches.
58–4 **4** inches.
58–5 **6** inches. . . . . .**58**( )

59. One base of a trapezoid is **4** feet greater than the altitude and the other is **2** feet greater than the altitude. Find the altitude of the trapezoid if its area is **54** square feet.

59–1 **8** feet.
59–2 **9** feet.
59–3 **12** feet.
59–4 **6** feet.
59–5 **10** feet. . . . . .**59**( )

60. From a point on the ground midway between two buildings, the angles of elevation of their tops are **30°** and **60°**, respectively. If the taller building is **60** feet high, what is the height of the other building?

60–1 **60** feet.
60–2 **20** feet.
60–3 **30** feet.
60–4 **40** feet.
60–5 **35** feet. . . . . .**60**( )

Number wrong	0 | 2	3 | 6	7 | 10	11 | 14	15 | 18	19 | 22	23 | 26	27 | 30	31 | 34	35 | 38	39 | 42	43 | 46	47 | +
Amount to be subtracted	0	1	2	3	4	5	6	7	8	9	10	11	12

Number right_____

Subtract (See table above) _____

Raw Score = Difference _____

OCCUPATIONAL INTEREST INVENTORY—Intermediate, Form A

Devised by Edwin A. Lee and Louis P. Thorpe

Name..... Sex: M—F..... Date.....

Company or School..... Position or Grade .....

City .....

1. .....

2. .....

FIELDS OF INTERESTS	Score	Percentile Rank	PERCENTILE (Chart percentile rank here) L | L-Av | H-Av | H — 1 10 20 30 40 50 60 70 80 90 99
A. P-S.			
B. Nat.			
C. Mech.			
D. Bus.			
E. Ar.			
F. Sci.			
TYPES OF INTERESTS			
1. Verb. . . . . (1)			
2. Manip. . . . . (2)			
3. Comp. . . . . (3)			
LEVEL OF INTERESTS			
a____ + d____ = ____ x 1 = ____			
b____ + e____ = ____ x 2 = ____			
c____ + f____ = ____ x 3 = ____			

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PART I.

DIRECTIONS: On this and the following pages you will find brief descriptions of kinds of work or things people do. You are to select the one of each pair that you would prefer if you had to choose one or the other. Do not consider how much you would earn, how much training would be necessary, or what people think of the activity. Assume that you must select one activity or the other even though, in some cases, you may not especially like either one or you may like both. Put a circle around the letter preceding the activity you choose.

Sample: A. Raise chickens, ducks, or turkeys and sell them.
B. Arrange a display of watches, rings, and other jewelry in a store window.

1

A Deliver groceries or meat to homes.

D[2] Wrap articles in the shipping department of a store.

2

F Work with foods, diets, and vitamins to solve nutrition problems.

A Investigate and solve crimes and lesser violations of the law.

3

E[2] Develop films or plates and print pictures.

D Address envelopes or fold letters and circulars for mailing.

4

A Call for and deliver laundry or clothing.

B[2] Grow flowers in a hot house for the market.

5

C Clean and oil small motors, vacuum cleaners, or bicycles.

D Type letters, bills, or statements for mailing.

6

C Build radio or television equipment.

E Design or construct stained glass, metal ornaments, or plastic art figures.

7

A Show people to their seats at theaters, games, or entertainments.

C Wash and grease automobiles or trucks.

8

B Raise pedigreed dogs, horses, or other animals.

C Operate lathes, drill presses, or planers.

9

D[2] File letters, bills, or reports in an office.

E Copy signs, posters, or campaign slogans.

10

E Draw plans for homes, public buildings, or apartment houses.

A[1] Help people to solve their personal problems.

11

C Patch and replace tubes and tires on wheels.

A Carry baggage or go on errands.

12

D[3] Write letters about prices, discounts, and sales commissions.

C Operate motion picture cameras.

13

E Improve the appearance of rock gardens, shrubbery, and flower beds.

F[2] Test and repair batteries, transformers, and other electrical equipment.

14

E[1] Conduct visitors through art galleries and museums.

C Help build automobiles, ships, or airplanes.

15

D Wrap bundles or tie packages for customers in a store.

B[2] Chop wood or split kindling for fuel.

(1____ 2____ 3____)

(A____ B____ C____ D____ E____ F____)

16

D Direct the sales policies for a large store or firm.

E[1] Write stories or articles for important magazines.

17

A Make beds, sweep floors, and do other housework.

C Do odd jobs with a saw, hammer, or plane.

18

E Sketch pictures, make etchings, or do free-hand drawings for pay.

B Help with the care and protection of valuable forests.

19

E[2] Clip hedges and trim trees.

C Mix cement, or carry plaster or bricks.

20

E[2] Paint pictures of landscapes, scenes, or flowers for exhibits.

F Carry out scientific experiments with chemicals.

21

B Take care of a flower or vegetable garden.

D Carry messages or deliver packages.

22

D[1] Show salesmen how to sell goods.

A Protect the rights and property of law-abiding people.

23

F Help a scientist care for his laboratory tools and equipment.

D[2] Unpack goods, keep a storeroom in order, or arrange articles on shelves.

24

A[1] Assist customers in selecting clothing, gifts, and other articles.

C Cut patterns in wood, metal, or plastics.

25

C Repair boots, shoes, or other footwear.

E Make simple articles of leather, reed, or beads.

26

F[3] Solve problems by the use of mathematics and make graphs illustrating the findings.

D Manage a large store or group of stores.

27

D[1] Answer a business telephone, or work at a small telephone switchboard.

A Deliver newspapers, advertising circulars, or other printed matter to homes.

28

E Draw plans for garden, lawn, or highway landscaping.

C Make models of boats, airplanes, or houses.

29

D Keep receipts or other records in order.

F Collect rocks, crystals, or other earth formations.

30

F Predict weather conditions and study the causes of storms.

C Plan or make labor-saving devices or equipment.

(1____ 2____ 3____)

(A____ B____ C____ D____ E____ F____)

31

E[2] Make hook rugs or bathroom mats.

B Catch fish or hunt wild animals for profit.

32

C Make drawings with a ruler, compass, or triangle.

E Arrange stage settings, or lighting and shadow effects.

33

A[1] Sell newspapers or magazines on the street.

D Sell candy, popcorn, or peanuts for profit.

34

D[3] Devise sales agreements, price sheets, and order forms.

B Trap or raise fur-bearing animals.

35

F[2] Prepare instruments and materials for scientific experiments.

C Label bottles, pack eggs, or wrap fruit.

36

D[3] Check the accuracy of financial statements or records.

F Use scientific laws to develop new machinery.

37

B[2] Sort or pack fruit or vegetables for the market.

E Make artificial flowers, fruit, or vegetables.

38

A[1] Teach children history, arithmetic, and other school subjects.

C Repair clocks, radios, or bicycles.

39

E Finish and decorate unpainted furniture or ornamental articles.

A[2] Shine people's shoes and brush their clothes.

40

A Help sick children or other members of a family to get well.

D[1] Take charge of the selection and placement of clerks or factory workers.

41

C Wax floors, wash windows, or dust woodwork.

F Help test milk, butter, cheese, or other dairy products.

42

F Test various kinds of minerals.

E Draw cartoons, comic strips, or caricatures.

43

F Help grow and keep records concerning unusual plants or flowers.

B Pick apples, berries, or tomatoes.

44

B Raise vegetables, flowers, or other garden products for profit.

E[1] Imitate the speech of well-known characters on stage or radio.

45

A Take care of young children while their parents are absent.

E Carve wood, stone, or metal ornamental figures.

(1____ 2____ 3____)

(A____ B____ C____ D____ E____ F____)

46

E Compose or arrange music for a chorus or orchestra.

D[1] Develop new ideas for advertising and selling goods.

47

F[3] Make simple graphs, charts, and tables of numbers.

A Wait on people at a soda fountain or candy store.

48

C Repair automobile motors and ignition systems.

F Test tooth-pastes and mouth-washes.

49

B Harvest wheat, beans, or rice.

F Remove paint or grease from wood and metal surfaces.

50

D[3] Manage the financial accounts and collections in a large company.

C Determine the strength of steel or concrete structures.

51

B Milk cows and feed and care for livestock.

A[3] Sell tickets at movie theaters, plays, or entertainments.

52

A Direct games and other activities on children's playgrounds.

E Arrange color harmonies, furniture combinations, and decorations.

53

A Keep an office tidy, run errands, and do odd jobs for an employer.

F Make face cream, soap, or tooth paste.

54

B Raise chickens, turkeys, or ducks for the market.

A Enforce health regulations and prevent the spread of disease.

55

C Operate a passenger or freight elevator.

B Herd cattle or sheep on the range.

56

B Prevent and treat diseases of horses, cattle, or hogs.

F[3] Use telescopes and mathematics to learn more about stars and planets.

57

A[2] Manicure fingernails and give shampoos.

B Destroy worms, insects, and other pests.

58

C Operate a printing press or a power sewing-machine.

B Raise cattle, sheep, or hogs for the market.

59

D[3] Keep the accounts and collect the money for a paper route.

C Assist a carpenter in building a house or a garage.

60

A[1] Work with courts and judges to help people get fair treatment.

F Analyze mineral formations.

(1___ 2___ 3___)

(A___ B___ C___ D___ E___ F___)

61

C Haul ashes, old cans, and waste paper.

B Cut down trees in a lumber camp.

62

A Coach baseball, basketball, tennis, or other sports.

F Fill medical prescriptions.

63

D Work in a store after school, weekends, or at Christmas time.

E[2] Make neckties, doilies, handkerchiefs, or tablecloths.

64

B Raise plants, bulbs, and seeds for the market.

A[1] Furnish information to travelers in a railroad station.

65

D Sell books, stationery, and other school supplies.

C Clean out offices, stores, or garages.

66

B Prevent and treat tree diseases.

F Experiment with flowers and plants to learn the laws governing their growth.

67

E Copy hand-painted Christmas or birthday cards, or party favors.

F Help to keep reports dealing with scientific observations.

68

E Design clothes, hats, or shoes.

B[2] Trim and prune trees or plants on a farm or in a nursery.

69

A Carry bags and show guests to rooms in a hotel.

E Knit scarfs, socks, or mittens.

70

D[3] Figure payrolls, salary rates, and salesmen's commissions.

E Conduct a band, orchestra, or chorus.

71

F Assist a druggist in labeling medicines, remedies, and other drugs.

E Set and shift scenery in a theatre.

72

C Paint or stain wood and metal surfaces.

D[3] Take inventories and keep records of goods in a small store.

73

F Assist in preparing dyes and cleaning fluids for use in a cleaning establishment.

E Paint and decorate drinking glasses, vases, and other glassware.

74

C Bake bread, pies, cakes, or rolls.

F[3] Measure the depth of oceans and the flow of ocean currents.

75

B Feed chickens and keep hen-coops clean.

D[3] Examine goods received to see if they agree with invoices.

(1____ 2____ 3____)

(A____ B____ C____ D____ E____ F____)

76

E Give public concerts on the piano or other instrument, or appear as a vocal soloist.

D[3] Study business trends, business costs, and methods of keeping down expenses.

77

C Help build roads or highways, or lay railroad tracks.

A Help keep passengers comfortable while traveling.

78

B Plant and care for trees, shrubs, or lawns.

E Play in an orchestra, band, or other musical organization.

79

C Do odd jobs and keep equipment in order in a store.

E Preserve flowers and shrubbery by the use of shellac or other preservatives.

80

F Develop substitutes for metal, rubber, wood, or glass.

A Help people to live better and more decent lives.

81

D Keep an office in order and do odd jobs for the "boss."

B Keep horse and cow stalls clean.

82

D[3] Keep records of production costs and overhead expenses in a factory.

F Prepare specimens on slides for study under a microscope.

83

D Buy and sell bicycles, radios, or other articles for profit.

F[3] Inspect the accuracy of scales in stores and butcher shops.

84

A Assist doctors in caring for sick or injured persons.

D[3] Make out monthly statements, balance sheets, and reports showing profit and loss.

85

E Make fancy candles or ornaments from tallow.

C Wash and iron clothes or launder shirts.

86

B Discover ways of increasing flower and plant production.

C Design and improve radios, diesel engines, or airplanes.

87

B[2] Clean and grade fruits and vegetables for the market.

C Whitewash fences, walls, sheds, or other surfaces.

88

F Check the effects of vitamins on white rats or other animals.

D[1] Sell goods to people in their homes or offices.

89

F Check the accuracy of gasoline pumps in filling stations.

D[3] Receive and check incoming supplies in a store.

90

C Improve cameras, telescopes, or microscopes.

B Plan better methods of mining gold, silver, and other valuable minerals.

91

B Cultivate beets, onions, or other vegetables.

E^2 Repair and refinish old furniture.

92

B Raise fish or frogs in private ponds for profit.

D^1 Prepare advertising circulars or business letters.

93

D Sell groceries or meats in a small-town store.

A Take orders and serve food to people in a restaurant.

94

B Manage a fish hatchery.

D Manage a shop, restaurant, or soda fountain.

95

F Maintain a constant temperature and humidity in a hot-house.

C Tighten bolts or solder wires.

96

D Direct the operation of a shipbuilding yard or oil company.

C Improve the lubrication and method of fueling of motor-driven machines.

97

E^2 Make simple vases and other objects out of clay.

B Clear land of weeds, brush, or stones.

98

A^1 Meet visitors and show people around an office or factory.

C Lay bricks or build fireplaces.

99

E Decorate and glaze dishes, pottery, or other earthenware.

D Deliver and collect money for C.O.D. packages.

100

C Design and construct bridges, tunnels, or mountain roads.

A Direct the affairs of a college or university.

101

F Take care of animals used in testing medicines or vitamins.

C Mend broken furniture, dishes, or ornaments.

102

E^1 Teach others to sing, play an instrument, or act.

F Breed animals or insects to learn about heredity.

103

B Feed pigs and care for their pens.

F Assist in the feeding and care of thoroughbred or pedigreed animals.

104

E Make pottery, statues, or book ends.

A^1 Advise regarding travel plans and sell tickets.

105

E^2 Paint waste baskets, clothes hampers, or wooden book ends.

B Raise flowers to sell.

(1____ 2____ 3____)

(A____ B____ C____ D____ E____ F____)

106

A Help to improve living conditions in cities or rural areas.

E[1] Write about the merits of concerts, radio programs, or operas.

107

A Help children to cross streets and to avoid traffic accidents.

F[3] Assist in keeping records of temperature and rainfall.

108

F Test methods for dyeing cloth and other materials.

A[2] Take temperatures and pulse rates, and make blood tests.

109

F[2] Fill bottles, paste labels, or wash test tubes.

B Know how to judge the good points of cattle, horses, or other animals.

110

C Invent tools for the manufacture of motors or other machinery.

B Develop methods of destroying pests such as worms, bugs, and other insects.

111

A Deliver milk, butter, eggs, or other dairy products.

B Produce milk and butter for the market.

112

E[1] Make announcements and take part in skits before a microphone.

F Operate x-ray machines.

113

F Sterilize dressings or medical instruments.

A Be a conductor on a street car or motor bus.

114

D Work up a small newspaper or magazine business.

F Test water or milk for impurities.

115

A Carry trays, wash dishes, or assist cooks in restaurants.

C Paint and refinish porch furniture, floors, or kitchen chairs.

116

F Prepare maps for geographies or encyclopedias.

B Develop new species of flowers or plants.

117

B Catch lobsters, shrimp, and other sea food, for the market.

A Drive a car and keep it in good condition for an employer.

118

D[2] Put labels on merchandise and place it on shelves.

B Work in a coal or other mine.

119

C Pump gasoline, fill radiators, and test batteries at a filling station.

D[3] Keep an account of the money received and spent in a small store.

120

B Develop new methods of soil conservation and flood control.

A Command a company or group of soldiers, sailors, or marines.

(1____ 2____ 3____)

(A____ B____ C____ D____ E____ F____)

PART II.

DIRECTIONS: Below you will find three activities under each number. You are to choose one of the three in each grade. Indicate your choice by a circle around the letter preceding the activity.

1

b. Advise regarding travel plans and sell tickets.
c. Help people to solve their personal problems.
a. Sell tickets at movie theaters, plays, or entertainments.

2

d. Cut down trees in a lumber camp.
e. Trim and prune trees or plants on a farm or in a nursery.
c. Prevent and treat tree diseases.

3

f. Build radio or television equipment.
d. Tighten bolts or solder wires.
b. Repair clocks, radios, or bicycles.

4

a. Sell candy, popcorn, or peanuts for profit.
f. Develop new ideas for advertising and selling goods.
b. Show salesmen how to sell more goods.

5

f. Paint pictures of landscapes, scenes, or flowers for exhibits.
e. Draw plans for garden, lawn, or highway landscaping.
a. Improve the appearance of rock gardens, shrubbery, and flower beds.

6

b. Prepare specimens on slides for study under a microscope.
d. Prepare instruments and materials for scientific experiments.
f. Carry out scientific experiments with chemicals.

7

e. Direct games and other activities on children's playgrounds.
f. Teach children history, arithmetic, and other school subjects.
d. Take care of young children while their parents are absent.

8

a. Take care of a flower or vegetable garden.
b. Grow flowers in a hot house for the market.
f. Develop new species of flowers or plants.

9

c. Design and improve radios, diesel engines, or airplanes.
a. Clean and oil small motors, vacuum cleaners, or bicycles.
e. Repair automobile motors and ignition systems.

10

a. Keep the accounts and collect the money for a paper route.
c. Manage the financial accounts and collections in a large company.
e. Figure payrolls, salary rates, and salesmen's commissions.

11

f. Design or construct stained glass, metal ornaments, or plastic art figures.
b. Make pottery, statues, or book ends.
d. Carve wood, stone, or metal ornamental figures.

12

a. Make simple graphs, charts, and tables of numbers.
e. Measure the depth of oceans and the flow of ocean currents.
c. Solve difficult problems by the use of mathematics and make graphs illustrating the findings.

13

e. Take temperatures and pulse rates, and make blood tests.
c. Help sick children or other members of a family to get well.
d. Help children to cross streets and to avoid traffic accidents.

14

a. Keep horse and cow stalls clean.
e. Raise pedigreed dogs, horses, or other animals.
c. Prevent and treat diseases of horses, cattle, or hogs.

(a____ b____ c____ d____ e____ f____)

15

c. Design and construct bridges, tunnels, or mountain roads.
d. Help build roads or highways, or lay railroad tracks.
e. Lay bricks or build fireplaces.

16

d. Keep receipts or other records in order.
f. Direct the sales policies for a large store or firm.
b. Devise sales agreements, price sheets, and order forms.

17

c. Draw plans for homes, public buildings, or apartment houses.
b. Sketch pictures, make etchings, or do free-hand drawings for pay.
a. Copy signs, posters, or campaign slogans.

18

e. Test various kinds of ores.
a. Collect rocks, crystals, or other earth formations.
c. Analyze the content of mineral formations.

19

b. Furnish information to travelers in a railroad station.
f. Help to improve living conditions in cities or rural areas.
d. Help keep passengers comfortable while traveling.

20

a. Clear land of weeds, brush, or stones.
b. Help with the care and protection of valuable forests.
c. Develop new methods of soil conservation and flood control.

21

c. Invent tools for the manufacture of motors or other machinery.
d. Pump gasoline, fill radiators, and test batteries at a filling station.
b. Help build automobiles, ships, or airplanes.

22

d. Work in a store after school, weekends, or at Christmas time.
f. Direct the operation of a shipbuilding yard or oil company.
e. Manage a shop, restaurant, or soda fountain.

23

f. Write about the merits of concerts, radio programs, or operas.
e. Make announcements and take part in skits before a microphone.
d. Set and shift scenery in a theater.

24

b. Test methods for dyeing cloth and other materials.
d. Make face cream, soap, or tooth paste.
c. Develop substitutes for metal, rubber, wood, or glass.

25

e. Protect the rights and property of law-abiding people.
c. Work with courts and judges to help people get fair treatment.
a. Deliver messages, go on errands, or help carry baggage.

26

a. Clean and grade fruits and vegetables for the market.
b. Raise vegetables, flowers, or other garden products for profit.
f. Discover ways of increasing flower and plant production.

27

c. Plan or make labor-saving devices or equipment.
d. Operate a passenger or freight elevator.
b. Operate a printing press or a power sewing-machine.

28

a. File letters, bills, or reports in an office.
f. Study business trends, business costs, and methods of keeping down expenses.
e. Keep records of production costs and overhead expenses in a factory.

29

f. Compose or arrange music for a chorus or orchestra.
b. Arrange color harmonies, furniture combinations, and decorations.
a. Make artificial flowers, fruit, or vegetables.

30

b. Breed animals or insects and keep records of their characteristics.
d. Help grow and keep records of unusual plants or flowers.
f. Experiment with flowers and plants to learn the laws governing their growth.

(a____ b____ c____ d____ e____ f____)

Name _____ Boy or Girl (Cross out one)
Last First Initial

Age today _____ Date of Birth _____
Years Months Year Month Day

Grade _____ Date _____

We would like your answers to the following questions. Be assured that your answers will have no bearing on your grades and will not become part of your record. Please take sufficient time to answer the questions honestly and frankly.

Answer each question by circling "Yes" or "No"

1. When you were in elementary school, did you dislike arithmetic?.....Yes No
2. When you were in junior high school, did you dislike mathematics?....Yes No
3. Do you dislike algebra?.....Yes No
4. Do you dislike geometry?.....Yes No
5. Do you like mathematics better than any other subject?.....Yes No

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